Week in Review # 14

MATH 142

8.1-8.3 Drost-Fall 2009

1. Evaluate the function

$$f(x,y) = \sqrt{8 - x - y^2}$$

a. $(2,1)$,

b. $(-4,2)$,

c. $(-1,0)$

1a. $f(2,1) = \sqrt{8-(2)-(1)^2}$

$f(2,1) = \sqrt{8-2-1} = \boxed{\sqrt{5}}$

1b. $f(-4,2) = \sqrt{8-(-4)-2^2}$

$f(-4,2) = \sqrt{8+4-4} = \sqrt{8} = \sqrt{4}\cdot\sqrt{2}$

$\boxed{2\sqrt{2}}$

1c. $f(-1,0) = \sqrt{8-(-1)-0^2}$

$f(-1,0) = \sqrt{8+1} = \sqrt{9} = \boxed{3}$

Find the domain of each of the following:

2. $f(x,y) = \dfrac{y}{x-2y}$

3. $f(x,y) = \sqrt{8-x+y}$

4. $f(x,y) = 3x^2\sqrt{y}$

5. $f(x,y) = \sqrt[3]{16+x^2+y}$

4. $y \geq 0$

5. $\mathbb{R}$

2. $x - 2y \neq 0$

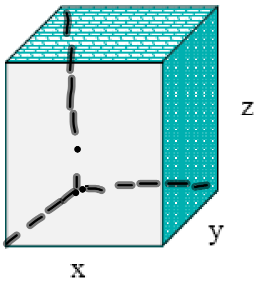
$x \neq 2y$

$\boxed{\frac{1}{2}x \neq y}$

3. $8 - x + y \geq 0$

$\boxed{y \geq x - 8}$

6. Find the surface area of a closed rectangular box whose volume is 1000 ft^3 as $S(x, y)$.



$V = x \cdot y \cdot z = 1000$

$z = \frac{1000}{xy}$

$S_A = 2xy + 2yz + 2xz$

$S_A = 2xy + 2\cancel{y}\left(\frac{1000}{x\cancel{y}}\right) + 2\cancel{x}\left(\frac{1000}{\cancel{x}y}\right)$

$S_A = 2xy + \frac{2000}{x} + \frac{2000}{y}$

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7. A company sells gadgets and widgets. The gadjets sell at $p = 250 - 6x - 4y$ and the widgets sell at $q = 800 - 2x - 5y$, where $x =$ the number of gadgets sold and $y =$ the number widgets sold. Find the revenue function, $R(x, y)$, and the value of $R(5, 6)$.

$R = x \cdot p + y \cdot q$

$R = x(250 - 6x - 4y) + y(800 - 2x - 5y)$

$R = 250x - 6x^2 - \underline{4xy} + 800y - \underline{2xy} - 5y^2$

$R = 250x - 6x^2 - 6xy + 800y - 5y^2$

$R(5,6) = 250(5) - 6(5^2) - 6(5)(6) + 800(6) - 5(6^2)$

$R(5,6) = 1250 - 150 - 180 + 4800 - 180$

$R(5,6) = 5540$

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Partial Derivatives: find f_x and f_y for each of the following functions.

8. $f(x,y) = 3x + 4y^2 - 2xy$
9. $f(x,y) = \sqrt{8 - x^2 - y^2}$
10. $f(x,y) = x^4 \cdot e^{2xy}$
11. $f(x,y) = \dfrac{x}{y^2 - 1}$
12. $f(x,y) = \ln\,(x^2 + 4y)$

8. $f_x = 3 - 2y$, $f_y = 8y - 2x$

9. $f(x,y) = (8 - x^2 - y^2)^{1/2}$

$f_x = \frac{1}{2}(8 - x^2 - y^2)^{-1/2}(-2x)$ $\qquad$ $f_y = \frac{1}{2}(8 - x^2 - y^2)^{-1/2}(-2y)$

$f_x = \dfrac{-x}{(8 - x^2 - y^2)^{1/2}}$ $\qquad$ $f_y = \dfrac{-y}{(8 - x^2 - y^2)^{1/2}}$

10. $f(x,y) = x^4 \cdot e^{2xy}$

$f_x = x^4(2y e^{2xy}) + e^{2xy}(4x^3)$

$f_x = 2x^3(e^{2xy})(xy + 2)$

$f_y = x^4 \cdot (2x)e^{2xy}$

$f_y = 2x^5 e^{2xy}$

11. $f(x,y) = \dfrac{x}{y^2 - 1}$

$f_x = \dfrac{(y^2-1)(1) - x(0)}{(y^2-1)^2} = \dfrac{y^2-1}{(y^2-1)^2} = \dfrac{1}{y^2-1}$

$f_y = \dfrac{(y^2-1)(0) - x(2y)}{(y^2-1)^2} = \dfrac{-2xy}{(y^2-1)^2}$

12. $f(x,y) = \ln(x^2 + 4y)$

$f_x = \dfrac{2x}{x^2 + 4y}$

$f_y = \dfrac{4}{x^2 + 2y}$

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Second Partial Derivatives: find the four second-order partial derivatives for each of the following functions.

13. $f(x,y) = 3x^2 + 4y^2 - 2x^2y^3$

14. $f(x,y) = e^{x-2y}$

13. $f_x = 6x - 4xy^3$ | $f_y = 8y - 6x^2y^2$

$f_{xx} = 6 - 4y^3$ | $f_{yx} = -12xy^2$

$f_{xy} = -12xy^2$ | $f_{yy} = 8 - 12x^2y$

14. $f_x = (1)e^{x-2y}$ | $f_y = -2e^{x-2y}$

$f_{xx} = e^{x-2y}$ | $f_{yx} = -2e^{x-2y}$

$f_{xy} = -2e^{x-2y}$ | $f_{yy} = 4e^{x-2y}$

15. $f(x,y)=\ln(2x+y^2)$

16. $f(x,y)=\left(5x+6y^2\right)^4$

15. $f_x=\dfrac{2}{2x+y^2}$ $\qquad f_y=\dfrac{2y}{2x+y^2}$

$f_{xx}=\dfrac{(2x+y^2)(0)-2(2)}{(2x+y^2)^2}$ $\qquad f_{yx}=\dfrac{(2x+y^2)(0)-2y(2)}{(2x+y^2)^2}$

$f_{xx}=\dfrac{-4}{(2x+y^2)^2}$ $\qquad f_{yx}=\dfrac{-4y}{(2x+y^2)^2}$

16. $f_x=4(5x+6y^2)^3\cdot(5)$ $\qquad f_y=4(5x+6y^2)^3(12y)$

$f_x=20(5x+6y^2)^3$ $\qquad f_y=48y(5x+6y^2)^3$

$f_{xx}=60(5x+6y^2)^2(5)$ $\qquad f_{yx}=144y(5x+6y^2)^2(5)$

$f_{xx}=300(5x+6y^2)^2$ $\qquad f_{yx}=720y(5x+6y^2)^2$

$f_{xy}=60(5x+6y^2)^2(12y)$ $\qquad f_{yy}=48y\left[3(5x+6y^2)^2(12y)\right]+\left[5x+6y^2\right]^3\cdot 48$

$f_{xy}=720y(5x+6y^2)^2$ $\qquad f_{yy}=1728y^2(5x+6y^2)^2+48(5x+6y^2)^3$

$f_{yy}=48\cdot 36y^2(\text{mess})^2+48(\text{mess})^3$

$f_{yy}=48(\text{mess})^2\left[36y^2+\text{mess}\right]$

$f_{yy}=48(5x+6y^2)^2(36y^2+5x+6y^2)$

$f_{yy}=48(5x+6y^2)^2(42y^2+5x)$

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17. Minimize the surface area in problem number 6.

$$S_A = 2xy + \frac{2000}{x} + \frac{2000}{y}$$

$$S_A(x,y) = 2xy + 2000x^{-1} + 2000y^{-1}$$

$$S_x = 2y - 2000x^{-2} = 2y - \frac{2000}{x^2} = 0$$

$$S_{xx} = \frac{4000}{x^3} \qquad 2y = \frac{2000}{x^2}$$

$$S_{xy} = 2 \qquad y = \frac{1000}{x^2}$$

$$S_y = 2x - 2000y^{-2} = 2x - \frac{2000}{y^2} = 0$$

$$S_{yx} = 2 \qquad 2x = \frac{2000}{y^2}$$

$$S_{yy} = \frac{4000}{y^3} \qquad x = \frac{1000}{y^2}$$

$$D = \frac{4000}{x^3}\left(\frac{4000}{y^3}\right) - 2^2 \qquad y = \frac{1000}{x^2} \; ; \; x = \frac{1000}{y^2}$$

$$y = \frac{1000}{\left(\frac{1000}{y^2}\right)^2}$$

$$D(10,10) = \frac{4000}{1000}\left(\frac{4000}{1000}\right) - 4$$

$$= 4(4) - 4 \qquad y \cdot \frac{1000^2}{y^4} = 1000$$

$$= 16 - 4 = 12 \qquad \frac{1000^2}{y^3} = 1000$$

$D>0$, $f_{xx}>0$

concave up $\qquad 1000^2 = 1000y^3$

$\therefore (10,10)$ is a rel min $\qquad 1000 = y^3$

$$10 = y$$

$$x = \frac{1000}{y^2} = \frac{1000}{10^2} = 10$$

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8. A company sells x logic games at a price $p = 120 - 2x - 3y$ and y super-duper deluxe logic games at a price of $q = 150 + 3x - 5y$.

a. Find the quantity they should sell of each product to maximize revenue.

b. Find the price of each product which maximizes revenue.

$$R = xp + yq = x(120 - 2x - 3y) + y(150 + 3x - 5y)$$

$$R = 120x - 2x^2 - 3\cancel{xy} + 150y + 3\cancel{xy} - 5y^2$$

$$R = 120x - 2x^2 + 150y - 5y^2$$

$$R_x = 120 - 4x = 0 \Rightarrow 4x = 120, \boxed{x = 30}$$

$$R_y = 150 - 10y = 0 \Rightarrow 10y = 150, \boxed{y = 15}$$

$R_{xx} = -4$ $\quad$ $R_{yx} = 0$

$R_{xy} = 0$ $\quad$ $R_{yy} = -10$

$$D = (-4)(-10) - 0^2 = 40 > 0$$

$R_{xx} < 0$ concave down

$\therefore$ $(30, 15)$ is a rel max

19. $f(x,y) = 4x^2+8xy+16y$, Find the critical points and label as a maximum, minimum or a saddle point.

$f_x = 8x + 8y = 0$

$8y = -8x$

$y = -x$

$f_y = 8x+16 = 0$

$8x = -16$

$x = -2$

critical point $(-2,2)$

$f_{xx} = 8$

$f_{xy} = 8$

$f_{yx} = 8$

$f_{yy} = 0$

$D = 8(0) - 8^2 = -64$

$D < 0$

$\therefore (-2,2)$ is a saddle point

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20. $f(x,y) = 2x^2 - 2x^2y + 6y^3$, Find the critical points and label as a maximum, minimum or a saddle point.

$f(x,y) = 2x^2 - 2x^2y + 6y^3$

$f_x = 4x - 4xy = 0$

$4x(1-y) = 0$

$x = 0,\ y = 1$

$f_y = -2x^2 + 18y^2 = 0$

$-2(x^2 - 9y^2) = 0$

$-2(x-3y)(x+3y) = 0$

$x = 3y\ ;\ x = -3y$

$(0,0), (3,1), (-3,1)$

$f_{xx} = (4-4y)$

$f_{xy} = (-4x)$

$f_{yx} = (-4x)$

$f_{yy} = (36y)$

$D = (4-4y)(36y) - (-4x)^2$

$D(0,0) = 4 \cdot 0 - 0 = 0$ no information

$D(3,1) = (4-4)(36) - (-12)^2$

$= -144$ (3,1) saddle point

$D(-3,1) = (4-4)(36) - (12)^2$

$= -144$ (-3,1) saddle point

21. $f(x,y) = 4x^2 \cdot e^{xy^2}$ Find the first order partial derivatives, f_x, f_y.

$$f(x,y) = 4x^2 \cdot e^{xy^2}$$

$$f_x = 4x^2\left(e^{xy^2}\right)\left(y^2\right) + e^{xy^2}(8x)$$

$$f_x = 4xe^{xy^2}\left[xy^2 + 2\right]$$

$$f_y = 4x^2(2xy)e^{xy^2}$$

$$f_y = 8x^3y \cdot e^{xy^2}$$

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22. $f(x,y) = \ln 3x^2y$ Find the first order partial derivatives, f_x, f_y.

$$f(x,y) = \ln|3x^2y|$$

$$f_x = \frac{\overset{2}{\cancel{6}}\cancel{x}\cancel{y}}{\cancel{3}x^{\cancel{2}}\cancel{y}} = \frac{2}{x} \qquad f_y = \frac{\cancel{3}\cancel{x^2}}{\cancel{3}\cancel{x^2}y} = \frac{1}{y}$$

23. Maddy sells two types of homemade preserves and the weekly demand and cost equations are:

$p = 150 - 5x + 10y$

$q = 275 + 15x - 4y$

$C(x, y) = 400 + 8x + 12y$

where x represents the weekly demand for jars of peach preserves, and y represents the weekly demand for jars of quamquat preserves, p is the price of the peach preserves, and q is the price of the quamquat preserves, and $C(x, y)$ is the cost function.

a. Find the revenue function, $R(x, y)$, and evaluate $R(20, 30)$.

b. Find the profit function for Maddy's peach and quamquat preserves, $P(x, y)$ and evaluate $P(20, 30)$.

$R = xp + yq$

$R = x(150 - 5x + 10y) + y(275 + 15x - 4y)$

$R = 150x - 5x^2 + 10xy + 275y + 15xy - 4y^2$

$R = 150x - 5x^2 + 275y - 4y^2$

$C = 400 + 8x + 12y$

a. $R(20,30) = 150(20) - 5(20^2) + 275(30) - 4(30^2)$

$= 3000 - 2000 + 8250 - 3600$

$= \$5650$

b. $P = R - C$

$P = 150x - 5x^2 + 275y - 4y^2 - (400 + 8x + 12y)$

$P = 142x - 5x^2 + 263y - 4y^2 - 400$

$P(20,30) = 142(20) - 5(20^2) + 263(30) - 4(30^2) - 400$

$P(20,30) = 2840 - 2000 + 7890 - 3600 - 400$

$= \boxed{4730}$

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24. Byfold Blinds, Inc has a productivity function defined by $f(x, y) = 120x^{.2}y^{.8}$, where x is the units of labor (in thousands), and y is the units of capital used (in thousands). Currently the company is using 25,000 units of labor and 30,000 units of capital. Find the marginal productivity of labor and the marginal productivity of capital.

$$f(x,y) = 120\,x^{0.2}\,y^{0.8}$$

$$f_x = 0.2(120)\,x^{-0.8}\,y^{0.8}$$

$$f_x(25,30) = 24(25^{-0.8})(30^{0.8})$$

$$= 27.76874412$$

$$f_x(25,30) = 27.77$$ marginal productivity of labor

$$f_y = 0.8(120)\,x^{0.2}\,y^{-0.2}$$

$$f_y = 96\,x^{0.2}\,y^{-0.2}$$

$$f_y(25,30) = 96(25^{0.2})(30^{-0.2})$$

$$f_y(25,30) = 92.56248038 = 92.56$$

marginal productivity of capital

5. For Byfold Blinds, Inc to maximize productivity, should labor or capital be increased?

Capital because the increase in production will be largr.