150 Lecture Notes - Section 9.1
Systems of Equations

Definition: A system of equations is a set of equations that involve the same variables.

To SOLVE a system of equations, means to find all solutions which will make all the equations true.

Method of Substitution:

1. Solve one of the equations for one of the variables
2. Substitute this value into the other equation.
3. Solve
4. Substitute answer in original equation.
5. Solve for the other variable.

Solve: $\begin{cases} x + 2y = 1 \\ 5x - 4y = -23 \end{cases}$

Solve for x in eq 1: Substitute into other equation

$x = 1 - 2y$

$$5(1-2y) - 4y = -23$$
$$5 - 10y - 4y = -23$$
$$5 - 14y = -23$$
$$-14y = -28 \qquad x = 1 - 2(2)$$
$$y = 2 \qquad x = -3$$

solution $(-3, 2)$

A total of \$6,000 is invested in two funds paying 7% and $6\frac{1}{2}$% simple interest. The yearly interest is \$402.50 . How much is invested in each fund.

$x =$ \$ invested at 7%

$y =$ \$ invested at $6\frac{1}{2}$%

$x + y = 6000$

$.07x + .065y = 402.50$

To solve by substitution: solve equation one for y: $y = 6000 - x$, then replace y in the second equation with $6000 - x$.

$$.07x + .065(6000 - x) = 402.50$$
$$.07x + 390 - .065x = 402.50$$
$$.005x = 12.5$$
$$x = 2500$$

\$2500 at 7%, and \$3500 at $6\frac{1}{2}$%

Solve: $\begin{cases} x^2 + y^2 = 25 \\ (x-8)^2 + y^2 = 41 \end{cases}$

$$x^2 + y^2 = 25$$
$$y^2 = 25 - x^2$$

$$(x-8)^2 + y^2 = 41$$
$$x^2 - 16x + 64 + 25 - x^2 = 41$$
$$-16x + 89 = 41$$
$$-16x = -48$$
$$x = 3$$

$$x^2 + y^2 = 25$$
$$9 + y^2 = 25$$
$$y^2 = 16$$
$$y = \pm 4$$

$(3, 4), (3, -4)$

Solve: $\begin{cases} -x + y = 4 \\ x^2 + y = 3 \end{cases}$

solve equation one for y: $y = x + 4$

substitute this value for y in equation two: $x^2 + x + 4 = 3$

$$x^2 + x + 1 = 0$$
$$x = \frac{-1 \pm \sqrt{1 - 4(1)(1)}}{2(1)}$$
$$x = \frac{-1 \pm \sqrt{3}i}{2}$$

therefore no real solutions

Elimination Method

Solve: $\begin{cases} 2x - y = 7.5 \\ x + 2y = 5 \end{cases}$

Solve: $\begin{cases} 3x^2 + 2y &= 14 \\ 7x^2 + 4y &= 32 \end{cases}$

Graphing Solutions

Solve: $\begin{cases} y &= \ln x \\ x + y &= 1 \end{cases}$

Graph both equations:

Solution : $(1, 0)$

A small business invests $10,000 in equipment to produce a product. Each unit costs 65 cents to produce and is sold for $ 1.20. How many items must be sold before the business breaks even?

The total cost is $(cost/unit)$ $(\#of\ units) + (initial\ cost)$

$C = 0.65x + 10,000$

The revenue is $(price/unit) \bullet (\#of\ units)$

$R = 1.20x$

Break-even point occurs when $R = C$

$1.20x = 0.65x + 10,000$

$.55x = 10,000$

$x = 18,181.81\overline{81}$

$x \approx 18,182$ units