Concepts to know # 2

MATH 150

Over Ch 3.1 thru 5.5

Drost-Fall '08

- 3.1 Polynomial Functions

  graph is smooth and continuous

  no breaks, holes, corners, cusps

  know: **end behavior**

  $x \to \infty$ or $x \to -\infty$

  even degree

  odd degree

  zeros of Polynomials

  $(x-c)$ is a factor $\to x = c$ is a root

  double roots: touch and turn

  a. Graph: $y = (x-a)(x+b)^2$

  b. Describe the end behavior:
  $y = (x-4)^2(x+1)^3(5-2x)$

  The number of local extrema, is at most one less than the degree.

  even functions: symmetric about y-axis

  $f(-x) = f(x)$

  odd functions: symmetric about the origin

  $f(-x) = -f(x)$

- 3.2 Dividing Polynomials

  a. Find the remainder:
  $$\frac{x^4 - x^3 + x^2 - x + 2}{x-2}$$

  b. Find the value of
  $P(-2)$ when $P(x) = x^3 + 2x^2 - 7$

  c. Is $x-1$ a factor of $x^{567} - 3x^{400} + x^9 + 2$?

  d. Find a polynomial of least degree with zeros at -2,3,5.

  e. Find a polynomial of least degree with zeros at -2,3,5, with 4 being the coefficient of x^2.

  f. Divide: $\dfrac{x^4 - 2x^2 + 7x}{x^2 - x + 3}$

- 3.3 Real Zeros of Polynomials - **Optional**

  a. List all the possible rational zeros of
  $P(x) = 6x^4 - x^3 - 32x^2 + 5x + 10$

  b. List the possible number of positive real zeros of P(x) as defined above.

  c. List the possible number of negative real zeros of P(x) as defined above.

  d. Find the smallest upper bound for P(x).

  e. Find the largest lower bound for P(x).

  f. Find all the zeros of the function P(x).

  g. Find all the zeros of:
  $R(x) = 8x^3 + 10x^2 - 39x + 9$

  h. Find all the zeros of:
  $S(x) = 6x^4 - 7x^3 - 8x^2 + 5x$

- 3.4 Complex Numbers

  a. Evaluate $(5-3i)(1+i)$

  b. Evaluate $\dfrac{5-i}{3+4i}$

  c. Evaluate $\sqrt{-3}\sqrt{-12}$

  d. Find all solutions: $4x^2 - 16x + 19 = 0$

- 3.5 Complex Zeros - **Optional**

  a. Find a polynomial with integer coefficients with zeros of 2 and $4-i$.

  b. Find a polynomial with zeros $2i$ and $3i$.

- 4.1 Exponential Functions

  know graph of $y = b^x$

  increasing if base $b > 1$

  decreasing if base $0 < b < 1$

  Domain: $\Re$ Range: $y > 0$

  HA: $y = 0$ VA: none

  Be able to simplify

  Know all of the following & how to use them:

  Compound interest
  $$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

  Continuously compounded
  $A = Pe^{rt}$

  Growth formula $P = P_0e^{rt}$ Decay formula $P = P_0e^{-rt}$

  a. The number of birds is limited by the type of habitat required for nesting. The population behaves according to the model:
  $n(t) = \dfrac{5600}{0.5 + 27.5e^{-0.044t}}$, where t is measured in years.

  1. Find the initial bird population.
  2. What size does the population approach as time goes on?

  b. If \$3675 is borrowed at $7\frac{3}{4}\%$, compounded monthly for 6 months, how much will be due?

  c. If my summer bonus check is invested at $9\frac{1}{2}\%$ continuously compounded, how long before the money triples in value?

- 4.2 Logarithmic Functions

  Definition: $\log_b x = y$ iff $b^y = x$

  Be able to graph $\log_b x = y$

  Always increasing

  Domain: positive Range: $\Re$

  HA: none VA: $y = 0$

  log understood base 10

ln understood base e

Properties

$\log_b 1 = 0$ $\qquad$ $\log_b b^x = x$

$\log_b b = 1$ $\qquad$ $b^{\log_b x} = x$

Be able to simpify

a. $\log_9 81$

b. $e^{\ln\sqrt{5}}$

c. $\ln\left(\frac{1}{e}\right)$

Be able to solve:

d. $\log_x 8 = \frac{3}{2}$

e. $\log_x 9 = 2$

f. $\log_4 \sqrt{2} = x$

Be able to find the domain:

g. $h(x) = \log_5(8 - 2x)$

h. $g(x) = \ln(x - x^2)$

i. $f(x) = \sqrt{x-2} - \log_5(10 - x)$

- 4.3 Laws of Logarithms

$\log AB = \log A + \log B$

$\log \frac{A}{B} = \log A - \log B$

$\log A^p = p \log A$

Be able to expand using the laws

Be able to write as a single log.

Change of base formula

$$\log_a x = \frac{\log_b x}{\log_b a}$$

a. Simplify: $\log_2 8^{33}$

b. Simplify: $\log_4 192 - \log_4 3$

c. Simplify: $\log \sqrt{.00001}$

- 4.4 Solving eq'n with logs & exp.

To solve exp.

try to get like bases

if you can't get like bases, isolate exp. then take the ln or log of both sides

To solve logs

Case 1: If only one log, isolate log then use def. of log to rewrite as an exponential and solve that way

Case 2: If $\log_b X = \log_b Y$ then $X = Y$

If more logs, use laws to rewrite into case 1 or 2.

a. Solve: $4(1 + 10^{5x}) = 9$

b. Solve: $e^{3-5x} = 16$

c. Solve: $\log_5 x + \log_5(x + 1) = \log_5 20$

d. Solve: $\log_2(\log_3(\log x)) = 1$

e. Solve: $e^{2x} - 3e^x - 40 = 0$

f. Solve: $\dfrac{10}{1 + e^{-x}} = 2$

- 4.5 Modeling with Exponential & Logarithmis Functions

Be able to do a word problem involving exp & logs.

Exponential growth: $n(t) = n_o e^{rt}$

Exponential decay: $n(t) = n_o e^{-rt}$

For problems with radioactive decay, with half-life h, $r = \dfrac{\ln 2}{h}$

- 5.1 The Unit Circle

t	Terminal Pt.
0	$(1, 0)$
$\frac{\pi}{6}$	$\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
$\frac{\pi}{4}$	$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
$\frac{\pi}{3}$	$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\frac{\pi}{2}$	$(0, 1)$

- 5.2 Trig Functions of Real Numbers

$\sin t = y$ $\qquad$ $\cos t = x$ $\qquad$ $\tan t = \dfrac{y}{x}$

$\csc t = \dfrac{1}{y}$ $\qquad$ $\sec t = \dfrac{1}{x}$ $\qquad$ $\cot t = \dfrac{x}{y}$

Signs of the trig functions

Quadrant	Positive Functions
I	all
II	sin, csc
III	tan, cot
IV	cos, sec

Reference angles: find the smallest angle to the x-axis, always positive

Even-Odd properties

$\sin(-t)$	=	$-\sin t$
$\cos(-t)$	=	$\cos t$
$\tan(-t)$	=	$-\tan t$

$\sin\theta = \dfrac{opp}{hyp}$ $\qquad$ $\cos\theta = \dfrac{adj}{hyp}$ $\qquad$ $\tan\theta = \dfrac{opp}{adj}$

$\csc\theta = \dfrac{hyp}{opp}$ $\qquad$ $\sec\theta = \dfrac{hyp}{adj}$ $\qquad$ $\cot\theta = \dfrac{adj}{opp}$

Know special angles

radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
degree θ	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Reciprocal Identities

$\sin\theta = \dfrac{1}{\csc\theta}$ $\cos\theta = \dfrac{1}{\sec\theta}$ $\tan\theta = \dfrac{1}{\cot\theta}$

$\csc\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities

1. $\sin^2\theta + \cos^2\theta = 1$

5.3 Graphs of sin/cos functions

To graph on calculator: use radian

Know the graphs of $\sin t$, and $\cos t$

Be able to find amplitude and period

General forms:

$y = a\sin(bx - c)$

$y = a\cos(bx - c)$

where $|a|$ is the amplitude, $\frac{2\pi}{b}$ is the period & $\frac{c}{b}$ is the phase shift

In factored form:

$y = a\sin b(x - \frac{c}{b})$

$y = a\cos b(x - \frac{c}{b})$

Or set the expression $(bx - c)$ equal to 0 and 2π and solve.

That gives you the begining and ending point of one complete cycle.

a. Find the phase shift of: $y = 4\sin(3x - \pi)$

b. Find the period of: $y = 3\sin(2x - \frac{\pi}{2})$

c. Find the amplitude of: $y = \frac{-1}{2}\cos(4x - \frac{\pi}{3})$

5.4 Graphs of other trig functions

Know the graphs of $\tan t$, $\cot t$, $\csc t$, $\sec t$

Be able to shift & reflect if needed.

Period of tan is π divided by the coefficient of x.

a. Find the period of $y = 4\tan(3x - \pi)$

b. Find the phase shift for y in part a.

5.5 Modeling Harmonic Motion - **Optional**

$y = a\sin(\omega t)$ or $y = a\cos(\omega t)$

amplitude $= \mid a \mid$

period $= \dfrac{2\pi}{\omega}$

frequency $= \dfrac{\omega}{2\pi}$

a. The displacement of a mass on a spring is modeled by $y = 4\sin(8\pi t)$ where y is measured in inches and t in minutes. find the amplitude, period, and the frequency.

b. A bolt is suspended from a spring. The bolt is compressed a distance of 3 inches and then released. The bolt returns to the compressed position in 2 sec. Find a function that models the displacement of the bolt.

Answers:

Section 3.1

a. Graph

b. as $x \to \infty, y \to -\infty$,
as $x \to -\infty, y \to -\infty$

Section 3.2

a. 12

b. $P(-2) = -7$

c. no

d. $P(x) = a(x+2)(x-3)(x-5)$

e. $P(x) = -\dfrac{2}{3}(x+2)(x-3)(x-5)$

f. $x^2 + x - 4 + \dfrac{12}{x^2 - x + 3}$

Section 3.3

a. $\dfrac{p}{q} \in \left\{\pm 1, 2, 5, 10, \dfrac{1}{2}, \dfrac{5}{2}, \dfrac{1}{3}, \dfrac{2}{3}, \dfrac{5}{3}, \dfrac{10}{3}, \dfrac{1}{6}, \dfrac{5}{6}\right\}$

b. 2 or 0

c. 2 or 0

d. 3

e. -3

f. $x = -\dfrac{1}{2}, \dfrac{2}{3}, \sqrt{5}, -\sqrt{5}$

g. $x = \dfrac{3}{2}, \dfrac{1}{4}, -3$

h. $x = 0, -1, \dfrac{5}{3}, \dfrac{1}{2}$

Section 3.4

a. $8 + 2i$

b. $\dfrac{11}{25} - \dfrac{23}{25}i$

c. -6

d. $2 \pm \dfrac{i\sqrt{3}}{2}$

Section 3.5

a. $P(x) = a(x^3 - 10x^2 + 33x - 34)$

b. $P(x) = x^2 - 5ix - 6$

Section 4.1

a. 1. 200 birds

a. 2. 11,200 birds

b. $\$3,819.73$

c. approx 11.6 yrs

Section 4.2

a. 2

b. $\sqrt{5}$

c. -1

d. 4

e. 3

f. $\frac{1}{4}$

g. $x < 4$

h. $(0, 1)$

i. $[2, 10)$

Section 4.3

a. 99

b. 3

c. $-\frac{5}{2}$

Section 4.4

a. $x = \frac{1}{5} \log \frac{5}{4}$

b. $x = \frac{3}{5} - \frac{1}{5} \ln 16$

c. $x = 4$

d. $x = 1,000,000,000$

e. $x = \ln 8$

f. $x = -\ln 4$

Section 5.3

a. $\frac{\pi}{3}$ to the right

b. π

c. $\frac{1}{2}$

Section 5.4

a. $\frac{\pi}{3}$

b. $\frac{\pi}{3}$ to the right

Section 5.5

a. amp = 4 inches, period = $\frac{1}{4}$ min,
freq = 4 cycles/min

b. $y = 3\cos(\pi t)$