Concepts to know # 3

MATH 150 Drost-Spring 2008

Over Ch. 6, 7 and 8

- 6.1 Angle Measure

  Be able to change from degrees to radians.

  Length of a circular arc: $s = r\theta$.

  Area of a circular Sector: $A = \frac{1}{2}r^2\theta$.

  Linear speed: $v = \frac{s}{t}$

  Angular speed: $\omega = \frac{\theta}{t}$

  Relationship between linear & angular speed: $v = r\omega$

- 6.2 Trigonometry of Right Triangles

  Know the definitions of all the trig functions

  Know the signs of the trig functions in each quadrant

  Reference angles

- 6.3 Trig Functions of Angles

  Must know the following:

  Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u} \quad \tan u = \frac{1}{\cot u}$$
$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u} \quad \cot u = \frac{1}{\tan u}$$

  Area of Triangle: $A = \frac{1}{2}ab\sin\theta$

- 6.4 Law of Sines: ASA, AAS, SSA

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

  Know what an oblique triangle is.

  Be able to use law of sines.

  Know ambiguous case (SSA), exist when $h < a < b$ where $h = b\sin A$

  Know how to find area of an oblique triangle.

  Area $= \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$

- 6.5 Law of Cosines: SSS, SAS

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ba\cos C$$

  Be able to use law of cosines.

- 7.1 Trigonometric Identities Identities

$$\sin(\tfrac{\pi}{2} - u) = \cos u \quad \cos(\tfrac{\pi}{2} - u) = \sin u$$
$$\tan(\tfrac{\pi}{2} - u) = \cot u \quad \cot(\tfrac{\pi}{2} - u) = \tan u$$
$$\sec(\tfrac{\pi}{2} - u) = \csc u \quad \csc(\tfrac{\pi}{2} - u) = \sec u$$

  Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

  Even/Odd Identities

$$\sin(-u) = -\sin u \quad \cos(-u) = \cos u$$
$$\tan(-u) = -\tan u \quad \csc(-u) = -\csc u$$
$$\sec(-u) = \sec u \quad \cot(-u) = -\cot u$$

  Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad \tan^2 u + 1 = \sec^2 u$$
$$\cot^2 u + 1 = \csc^2 u$$

- 7.1 Verifying Trig Identities

  Usually start with more complicated side

  Change to sin and cos when possible

  If nothing works, try multiplying by conjugates

  PRACTICE, PRACTICE, PRACTICE

- 7.2 Sum and Difference Formulas

  Must know sum and difference formulas:

$$\sin(A + B) = \sin A\cos B + \cos A\sin B$$
$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$
$$\cos(A + B) = \cos A\cos B - \sin A\sin B$$
$$\cos(A - B) = \cos A\cos B + \sin A\sin B$$
$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$
$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

  Be able to use sum & difference formulas to find exact values of sin, cos, & tan.

  Be able to verify equations involving sum/differences.

- 7.3 Multiple-Angle & Product-Sum Formulas

  Must know Double-Angle formulas:

$$\begin{aligned}\sin 2A &= 2\sin A\cos A\\ \cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\\ \tan 2A &= \frac{2\tan A}{1 - \tan^2 A}\end{aligned}$$

  Be able to simplify expressions & solve equations involving double-angles

  Be able to use Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2} \qquad \cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

  Be able to use Half-Angle Formulas

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1 - \cos A}{2}}$$
$$\cos\frac{A}{2} = \pm\sqrt{\frac{1 + \cos A}{2}}$$
$$\tan\frac{A}{2} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}$$

  The sign is determined by which quadrant $\frac{A}{2}$ is in.

  Be able to use Product-to-Sum Formulas

$$\sin A \cdot \sin B = \tfrac{1}{2}[\cos(A - B) - \cos(A + B)]$$
$$\cos A \cdot \cos B = \tfrac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$\sin A \cdot \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$\cos A \cdot \sin B = \frac{1}{2}[\sin(A+B) - \sin(A-B)]$

Be able to use Sum-to-Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

- 7.4 Inverse Trig. Functions

$\sin^{-1} x = y \iff \sin y = x$

$\sin(\sin^{-1} x) = x$, $[-1, 1]$

$\sin^{-1}(\sin x) = x$, $[-\pi/2, \pi/2]$

$\cos^{-1} x = y \iff \cos y = x$

$\cos(\cos^{-1} x) = x$, $[-1, 1]$

$\cos^{-1}(\cos x) = x$, $[0, \pi]$

$\tan^{-1} x = y \iff \tan y = x$

$\tan(\tan^{-1} x) = x$, $\Re$

$\tan^{-1}(\tan x) = x$, $[-\pi/2, \pi/2]$

- 7.5 Trig Equations

isolate the trig function, use algebra to solve

sometime, factoring is useful

use substitution when necessary to write the expression in terms of only one trig function

if you square both sides, check solutions for extraneous roots

- 8.4 Vectors

Know what a vector is, standard position, zero vector, unit vector

Be able to write in component form given endpoints of line segment $\vec{PQ}$

$\mathbf{v} = < v_1, v_2 > = < q_1 - p_1, q_2 - p_2 >$

Given $\mathbf{v} = < v_1, v_2 >$, $||\mathbf{v}|| = \sqrt{v_1^2 + v_2^2}$

$\mathbf{v} = ||\mathbf{v}|| < \cos\theta, \sin\theta > = ||\mathbf{v}|| \cos\theta \mathbf{i} + ||\mathbf{v}|| \sin\theta \mathbf{j}$

Be able to find component and magnitude of resultant vector given two vectors and the angle between them.

Be able to do scalar multiplication & vector addition.

Be able to find a unit vector $\mathbf{u} = \dfrac{\mathbf{v}}{||\mathbf{v}||}$ in the direction of $\mathbf{v}$

Be able to find direction angle of vector $\mathbf{v}$

Be able to find angle between forces F_1 & F_2 given the magnitude of their resultant.

- 8.5 Dot Products

Know def. of dot product

$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2$

Be able to find angle btwn 2 vectors

$\cos\theta = \dfrac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{u}||\,||\mathbf{v}||}$

Know what orthogonal means.

Be able to decompose a vector into components which are orthogonal.

Know how to find projection of $\mathbf{u}$ onto $\mathbf{v}$

$proj_{\mathbf{v}}\mathbf{u} = \left(\dfrac{\mathbf{u} \cdot \mathbf{v}}{||\mathbf{v}||^2}\right)\mathbf{v}$

And for the **Final Exam** review also

- 9.1 Solving Systems of Equations

Graphing (calculator)

Substitution method

Be able to solve using Addition/Subtraction method (Method of Elimination)

Know consistent has at least one solution & inconsistent has no solution.

- 9.9 Systems of Inequalities

Be able to graph.

Determine if solution is bounded or unbounded.

Given a graph, determine a set of inequalities

- 10.1 Parabolas

Know basic formulas for parabola with:

a) vertical axis $(x-h)^2 = 4p(y-h)$

b) horizontal axis $(y-h)^2 = 4p(x-h)$

Be able to find the **focal diameter** of the parabola.

- 10.2 Ellipses

Basic ellipse (horiz major axis) $a > b > 0$

$$\frac{(x-h)^2}{a^2} + \frac{(y-h)^2}{b^2} = 1$$

Basic ellipse (vertical major axis) $a > b > 0$

$$\frac{(x-h)^2}{b^2} + \frac{(y-h)^2}{a^2} = 1$$

Be able to find the vertices, foci, and graph.

Eccentricity of an Ellipse: $e = \dfrac{c}{a}$ where

$c = \sqrt{a^2 - b^2}$.

The closer e is to zero, the ellipse is more "circular". The eccentricity of every ellipse ranges from $0 < e < 1$.

- 10.3 Hyperbolas

  Basic hyperbola (horiz transverse axis)

  $$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

  Basic hyperbola (vertical transverse axis)

  $$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

  Be able to find the asymptotes, foci, and graph.