Week in Review # 5

MATH 150 eby-Fall 2002

3.1 through 3.3 drost-Fall 2002

Section 3.1

1. Sketch the graph of $f(x) = (2x-1)(x+1)(x+3)^2$.

2. Sketch the graph of $f(x) = (x-a)^2(x+b)$

3. Describe the end behavior of:
   $g(x) = -4x^5 + 3x^2 + 5$

4. Describe the end behavior of:
   $h(x) = -2x^4 + 3x^2 - 1$

5. An open box is constructed from a piece of cardboard 20 cm x 40 cm by cutting squares of length x from each corner and folding up the sides.
   a. Express the volume as a function of x.
   b. What is the domain?

Section 3.2

6. Find the quotient and the remainder using long division for
   $$\frac{x^3 + 3x^2 + 4x + 3}{3x + 6}$$

7. Find the quotient and the remainder using long division for
   $$\frac{3x^3 - 12x^2 - 9x + 1}{x - 5}$$

8. Evaluate $P(\frac{1}{2})$ given
   $P(x) = 2x^2 + 9x + 1$

9. Evaluate $P(-7)$ given
   $P(x) = 5x^4 + 30x^3 - 40x^2 + 36x + 14$

10. Show that $(x - 2)$ is a factor of $P(x) = x^3 + 2x^2 - 3x - 10$

11. Find a polynomial of degree 4 with zeros at -2, 0, 2, and 4, which passes through the point $(1, -27)$.

12. Find a polynomial of degree 3 with zeros at 1, -2, and 3 with the coefficient of x^2 being $\frac{-1}{3}$.

13. Find a polynomial of least degree with zeros at 4 and $2i$.

14. Find a polynomial with integer coefficients with zeros at 5 and $4 + 2i$.

Section 3.3

Given $P(x) = 3x^5 - 14x^4 - 14x^3 + 36x^2 + 43x + 10$

15. Find the sum of the possible positive ration roots for $P(x)$ as defined above.

16. Find the number of positive real zeros of $P(x)$ as defined above.

17. Find the number of negative real zeros of $P(x)$ as defined above.

18. Find an upper and lower bound for $P(x)$ as defined above.

19. Find all the zeros of the function $P(x)$ as defined above.

$P(x) = 2x^6 - 3x^5 - 13x^4 + 29x^3 - 27x^2 + 32x - 12$

20. Find all the zeros of $P(x)$.

$P(x) = 4x^4 - 21x^2 + 5$

21. Find all the zeros of $P(x)$.

22. Given the volume is 1500 cm^3 for the box problem (# 5), find the value(s) of x which produces this volume.

ANSWERS:

3. as $x \Rightarrow -\infty, y \Rightarrow \infty$
   as $x \Rightarrow \infty, y \Rightarrow -\infty$

4. as $x \Rightarrow -\infty, y \Rightarrow -\infty$
   as $x \Rightarrow \infty, y \Rightarrow -\infty$

5. a) $V = 4x^3 - 120x^2 + 800x$
   b) domain $(0, 10)$

6. quotient: $\frac{1}{3}x^2 + \frac{1}{3}x + \frac{2}{3}$; remainder -1

7. quotient: $3x^2 + 3x + 6$; remainder 31

8. $P(\frac{1}{2}) = 6$

9. $P(-7) = -483$

10. x-2 is a factor since $P(2) = 0$

11. $P(x) = -3(x+2)(x)(x-2)(x-4)$

12. $P(x) = \frac{1}{6}x^3 - \frac{1}{3}x^2 - \frac{5}{6}x + 1$

13. $P(x) = x^2 - (4+2i)x + 8i$

14. $P(x) = x^3 - 13x^2 + 60x - 100$

15. 24

16. 2

17. 3

18. 6 upper bound; -2 lower bound

19. zeros $2, 5, -\frac{1}{3}, -1$(mult 2)
20. zeros $-3, \frac{1}{2}, 2$(mult 2), $i, -i$
21. zeros $-\frac{1}{2}, \frac{1}{2}, \sqrt{5}, -\sqrt{5}$
22. $x = 5, 3.486$ cm.