PRACTICE FINAL I

1. Find a general solution of $\mathbf{x}'(t) = A\mathbf{x}(t)$.

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix},$$

2. Find a general solution of $\mathbf{x}'(t) = A\mathbf{x}(t) + \mathbf{f}(t)$.

$$A = \begin{bmatrix} -4 & 2 \\ 2 & -1 \end{bmatrix},$$

$f(t) = (t^{-1}; 4 + 2t^{-1})$.
3. Solve $\frac{dy}{dx} + 4y - e^{-x} = 0$, $y(0) = 4/3$.
4. Solve $y''' - y'' - 4y' + 4y = 0$, $y(0) = -4$, $y'(0) = -1$, $y''(0) = -19$.
5. Determine inverse Laplace transform of $\frac{1}{(s-2)^4} + \frac{5s^2+10s+11}{(s^2+2s+4)(s+1)}$
6. Solve initial value problem $y'' + 2y' + 2y = \delta(t-1) - \delta(t-2)$, $y(0) = 2$, $y'(0) = 2$.
7. Using the method of Laplace transform solve:

$$x' + y = 1 - u(t-2), \quad x + y' = 0, \quad x(0) = y(0) = 0.$$

8. Find a general solution $y'' + 4y = \tan(2t)$.
9. Find a general solution of the system $x'' + y'' - x' = 2t$, $x'' + y' - x + y = -1$.
10. Find Laplace transform (using convolution) of $(s+1)/(s^2+1)^2$.
11. Solve $y'' + 4y = g(t)$, $y(0) = 1$, $y'(0) = 3$, $g(t) = \sin(t)$ if $0 \le t \le 2\pi$ and $g(t) = 0$ if $t > 2\pi$.