Problem 1

Objective is to model numerically store business. Divide the customers into three groups: (1) people inside the store (M); (2) people waiting in line (q); (3) people who have already left the store (N). Assume that there is one line and R=4 cashiers. Further assume "average shopping time for all customers" is $\frac{1}{\lambda} = 2$ minutes, and it is distributed exponentially, and average check out time is $\frac{1}{\mu} = 0.2$ minutes and exponentially distributed. Store hours from 6am-10pm(=22). The customers enter the store with variable rate (though uniformly). You can take the following rates (customer per unit time (say minute), not necessarily integer number); $n_{non-peak}$ customers from 6-10am; n_{peak} customers from 10am-1pm; $n_{non-peak}$ customer from 1pm-4pm; n_{peak} customers from 4pm-7pm; $n_{non-peak}$ customers from 7pm-10pm.

1) Plot M, N, q as a function of actual time for different values of $n_{non-peak}$ and n_{peak}.

3) Use variable R (number of cashiers) for peak and non-peak times.

3)(optional) Assume that customers leave the store with probability 0.5 without purchasing anything if q is larger than some critical q (say 10).

Problem 2

Use rejection method to simulate 1000 random variable with probability distribution

$$f(x) = \frac{1}{2\sqrt{\pi}} \exp(-x^2/2)$$

on the interval $[0, \infty]$ using $g(x) = \exp(-x)$. Calculate the numerical cumulative distribution of the simulated random variables by counting how many of them are less than a given number. Plot the numerical cumulative distribution against analytical one. Use erf function for the analytical distribution.