

Final Exam Practice

These are sample problems similar to what you might find on the Final.

Instructions: *Show all of your work. Answers without sufficient justification will receive little or no credit. If you use maple to solve a problem then state so and write the commands you used.*

1. *Compute the Laplace transform of the following functions*

(a) $\sin(2t) \cos(3t)$

(b) $te^{-3t} \sin(2t)$

(c)

$$f(t) = \begin{cases} t^2 e^t & \text{if } t < 1, \\ \cos^2(t) & \text{if } t > 4 \\ 0 & \text{otherwise} \end{cases}$$

(d) *The periodic function $f(t)$ with period $T = 4$ for which f_T is given by*

$$f_T(t) = \begin{cases} t & \text{if } 0 \leq t \leq 2, \\ 4 - t & \text{if } 2 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

(e) *The periodic function $f(t)$ with period $T = 2\pi$ for which f_T is given by*

$$f_T(t) = \begin{cases} \sin(t) & \text{if } 0 \leq t \leq \pi, \\ 0 & \text{if } \pi \leq t \leq 2\pi \end{cases}$$

2. *Compute the Laplace transform of the following functions using Taylor series or the Gamma function*

(a) $f(t) = e^{2t^2}$

(b) $f(t) = 2 \sin(t^2) \cos(t^2)$

(c) $f(t) = t^{1/3}$

3. Compute the inverse Laplace transform of the following functions

(a) $F(s) = \frac{2s+5}{(s+1)^3(s^3-1)}$

(b) $F(s) = \frac{P(s)}{(s+1)(s+2)^2(s^2+2)(s^2+1)}$, where $\deg(P(s)) \leq 6$

(c) $F(s) = \frac{P(s)}{(s^4+s^2-2)(s^2+s-2)}$, where $\deg(P(s)) \leq 5$

4. Solve the following initial value problems using Laplace transforms

(a) $y'' - 7y' + 10y = 9 \cos(t) + 7 \sin(t)$, $y(0) = 5$, $y'(0) = -4$

(b) $y'' + 3ty' - y = 1$, $y(0) = 0$, $y'(0) = 0$

(c) $y''' + 4y'' + y' - y = -12$, $y(0) = 1$, $y'(0) = 4$, $y''(0) = -2$

5. Let A be a matrix

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$$

(a) Express $\det(A)$ in terms of 2×2 determinants.

(b) State a condition that will guarantee the existence of a solution for

$$A\mathbf{x} = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$$

(c) State the defining property of the inverse of A

6. Write the following systems of differential equations in normal form $\mathbf{x}' = A\mathbf{x} + \mathbf{f}$

(a)

$$\begin{aligned} \frac{d^2y}{dt^2} &= x + \frac{dx}{dt} \\ \frac{d^2x}{dt^2} + y &= \cos(t) \end{aligned}$$

(b)

$$\frac{d^3y}{dt^3} - \frac{dx}{dt} + e^t y = \sin(t + \pi) \frac{d^2x}{dt^2} = y$$

7. Compute the eigenvalues, eigenvectors, and inverses for the following matrices

(a)

$$\begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(c)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$$

8. Find a fundamental matrix $X(t)$ for the system $\mathbf{x}' = \mathbf{A}\mathbf{x}$ for each A

(a)

$$A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$$

(b)

$$A = \begin{pmatrix} 2 & -5 \\ 0 & 1 \end{pmatrix}$$

(c)

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$