STRAIGHT LINES AND LINEAR FUNCTIONS

1. The manufacture of \(x\) thousands of bolts has a fixed cost of $546 and variable cost of $221 per thousand. Revenue is $357.50 per thousand of bolts sold. Find and interpret the break-even point.

2. The supply and demand for cookies at a coffee shop are given by \(S(x) = p = 3x + 7\) and \(D(x) = 84 - 4x\) where \(p\) is the price in yen (¥) for a cookie and \(x\) is the number of cookies. Find and interpret the equilibrium point.

3. Find the slope and equation of the line that passes through the origin and
   (a) \((a,0)\) where \(a \neq 0\)
   (b) \((0,b)\) where \(b \neq 0\)

4. An asset costs $45,000 and has a 10 year life span. The scrap value of this asset is $5000. Find the equation for the linear depreciation of this asset and the rate of depreciation.

5. A discount store finds that it sells, on a daily basis, 10 ClickMe cameras when the price is $30 and they sell 6 when the price is $40. Find a demand equation for ClickMe cameras.

6. You are given the following data where \(y\) is the company's profit (in thousands of dollars) after \(x\) years,

\[
\begin{array}{c|cccc}
 x & 1 & 3 & 5 & 7 \\
 y & 100 & 130 & 170 & 220 \\
\end{array}
\]

Find the linear least squares equation. Use this least squares equation to estimate the company's profit after 10 years and find when the company has a profit of $150,000.

SYSTEMS OF LINEAR EQUATIONS AND MATRICES

1. A cash register has 95 bills in one-dollar, five-dollar and ten-dollar denominations. If there are twice as many five-dollar bills as ten-dollar bills and all the bills together are worth $350, how many of each kind of bill are there?

2. A merchant wishes to mix three kinds of coffee, Columbian (selling for $8 per pound), Kona ($10 per pound) and Blue Mountain ($15 per pound) to get 50 pounds of a mixture that can be sold for $11.70 per pound. The amount of the Columbian must be 3 pounds more than the amount of Kona. Find the number of pounds of each that will be used.

3. Solve the following system of linear equations:
   \[
   \begin{align*}
   3x + 2y - z &= 0.5 \\
   -1.5x - y + 0.5z &= -0.25 \\
   x + 2y + z &= -1
   \end{align*}
   \]

4. Solve the following system using the Gauss-Jordan method:
   \[
   \begin{align*}
   2x + 3y - z &= 10 \\
   5x - y + 2z &= 19
   \end{align*}
   \]
5. A trucking firm wants to purchase 11 trucks that will provide exactly 31 tons of additional shipping capacity. A model A truck holds 2 tons, a model B truck holds 3 tons and a model C truck holds 6 tons. How many trucks of each model should the company purchase to provide the additional shipping capacity?

6. Answer the following as true or false.
   a) Every matrix can be squared.  
   b) Every square matrix has an inverse.  
   c) Every matrix has a transpose.  
   d) If the product $AB$ exists then the product $BA$ exists.  
   e) $A + A^{-1} = I$

7. Solve this matrix equation for $a$, $b$, $c$, and $d$:  
   \[
   \begin{bmatrix}
   3 & 2 \\
   0 & -1
   \end{bmatrix} + \begin{bmatrix}
   a & b \\
   c & d
   \end{bmatrix} \begin{bmatrix}
   0 & 4^T \\
   3 & 1
   \end{bmatrix} = \begin{bmatrix}
   5 & 2 \\
   1 & 0
   \end{bmatrix}
   \]

8. The grams of fat (F), mg. of sodium (S) and units iron (I) per serving of beans, meat and squash is given by $A = \begin{bmatrix}
   \text{beans} & \text{meat} & \text{squash} \\
   2 & 5 & 1 \\
   8 & 3 & 2 \\
   4 & 6 & 0
   \end{bmatrix}$

If 2 servings of beans, 1.5 servings of meat and 3 servings of squash are eaten, find a matrix $B$ that when multiplied by $A$ will give the total intake of fat, sodium and iron.

9. Solve the following matrix equation for $B$:  
   $AB - B = C$ .

10. A supermarket chain sells oranges and apples in two different stores. The average number of pounds sold per day in each store is in matrix $M$. In matrix $N$ is the "in season" and "out of season" prices, in dollars per pound, for each fruit. What does the product matrix $MN$ represent?

<table>
<thead>
<tr>
<th></th>
<th>oranges</th>
<th>apples</th>
<th>in season</th>
<th>out of season</th>
</tr>
</thead>
<tbody>
<tr>
<td>store 1</td>
<td>95</td>
<td>90</td>
<td>0.50</td>
<td>1.25</td>
</tr>
<tr>
<td>store 2</td>
<td>50</td>
<td>45</td>
<td>0.40</td>
<td>1.50</td>
</tr>
</tbody>
</table>

11. An economy consists of three sectors: mining ($M$), farming ($F$), and tourism ($T$).

<table>
<thead>
<tr>
<th></th>
<th>$M$</th>
<th>$F$</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>0.1</td>
<td>0.3</td>
<td>0.25</td>
</tr>
<tr>
<td>$A = F$</td>
<td>0.4</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>$T$</td>
<td>0.15</td>
<td>0.1</td>
<td>0.15</td>
</tr>
</tbody>
</table>

(a) For the input-output matrix $A$, interpret the meaning of entry $a_{23}$.

(b) If there is an external demand for 200 units of mining, 500 units of farming and 100 units of tourism, how much mining, farming and tourism needs to be produced to meet internal and external demands?