MARKOV CHAINS

1. Classify the following matrices as
(A) regular transition matrix (B) not regular transition matrix (C) not a transition matrix

_____ $\begin{bmatrix} 0 & 0.4 \\ 1 & 0.6 \end{bmatrix}$ _____ $\begin{bmatrix} 0.5 & 0.8 \\ 0.5 & 0.6 \end{bmatrix}$ _____ $\begin{bmatrix} 0.75 & 0 \\ 0.25 & 1 \end{bmatrix}$

2. A study has shown that a family living in the state of Denial typically takes a vacation once per year. A family that takes an out-of-state vacation has a 35% chance of taking an out-of-state vacation the following year and a 65% chance of taking an in-state vacation. A family that has taken an in-state vacation has a 50% chance of taking an out-of-state vacation the next year and 50% chance of taking an in-state vacation.
(a) What is the transition matrix for the vacation decision?
(b) If the initial distribution of vacations is 25% in-state and 75% out-of-state and people, what is the probability that a family will take an out-of-state vacation in two years?
(c) What is the long term distribution of vacation locations?

KEY

1. (A) because T^100 has only positive entries (no zeros)
(C) because column 2 adds to 1.4 so it can't be a transition matrix
(B) because T^100 has zeros (an absorbing markov process)

2. (A) $T = \begin{matrix} \\ out \\ in \end{matrix} \begin{matrix} \begin{matrix} out & in \end{matrix} \\ \begin{bmatrix} 0.35 & 0.50 \\ 0.65 & 0.50 \end{bmatrix} \end{matrix}$

(B) $X_0 = \begin{matrix} out \\ in \end{matrix} \begin{bmatrix} 0.75 \\ 0.25 \end{bmatrix}$, $X_2 = T^2 X_0 = \begin{matrix} out \\ in \end{matrix} \begin{bmatrix} 0.441875 \\ 0.558125 \end{bmatrix}$

$$X_L = TX_L \quad \Leftrightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.35 & 0.50 \\ 0.65 & 0.50 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Leftrightarrow \begin{matrix} x = 0.35x + 0.50y \\ y = 0.65x + 0.50y \end{matrix}$$

(C) $\Leftrightarrow \begin{matrix} 0 = -0.65x + 0.50y \\ 0 = 0.65x - 0.50y \end{matrix}$ and $x + y = 1$

$$\left[\begin{array}{cc|c} -.65 & .5 & 0 \\ .65 & -.5 & 0 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 0 & 10/23 \\ 0 & 1 & 13/23 \\ 0 & 0 & 0 \end{array}\right]$$

$x \approx 0.4348$ out of state
$y \approx 0.5652$ in of state

