

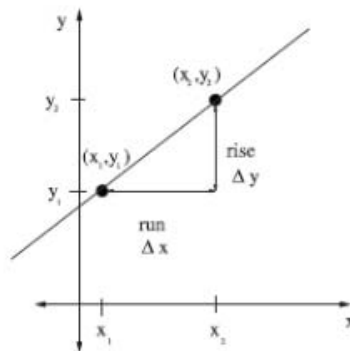
<http://www.math.tamu.edu/~epstein/FiniteMath/>

WEEK 1 REVIEW – Lines and Linear Models

SLOPE

A VERTICAL line has NO SLOPE. All other lines have

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Example

Find the slope of the line passing through the points $(-2, 4)$ and $(0, -4)$

Answer

Let one pair of points be (x_1, y_1) and the other (x_2, y_2) . Then

EQUATIONS OF LINES

The formula for the slope of a line can be rearranged to give us the equation for a line.

$$m = \frac{y - y_1}{x - x_1} \rightarrow y - y_1 = m(x - x_1)$$

This is called the POINT-SLOPE form of a line. If you know a point, (x_1, y_1) that lies on the line and you know the slope, m , of the line, then you can find the equation of the line.

Example

What is the equation of the line passing through the points $(-2, 4)$ and $(0, -4)$?

Answer

$m = -4$ (previous example) Let $(x_1, y_1) = (-2, 4)$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = (-4)(x - (-2))$$

$$y - 4 = -4x + (-4)(2)$$

$$y + 4 = -4x - 8 + 4$$

$$y = -4x - 4$$

if $(x_1, y_1) = (0, -4)$

$$y - y_1 = m(x - x_1)$$

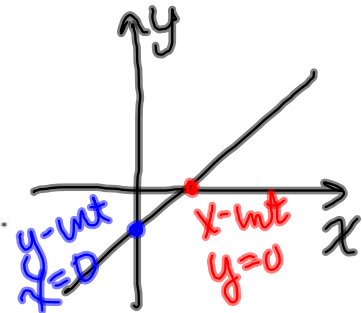
$$y - (-4) = (-4)(x - 0)$$

$$y + 4 = -4x \Rightarrow y = -4x - 4$$

When we simplify our point-slope form we are writing the line in the slope-intercept form,

$$y = mx + b$$

Again, m is the slope and now b is the y -intercept.



The y -intercept is the place where the line crosses the y -axis.
The x -intercept is the place where the line crosses the x -axis.

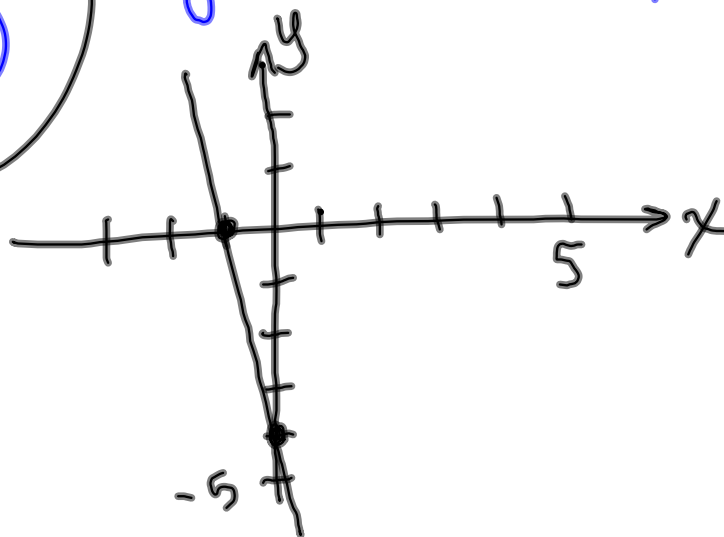
Example

Graph the line $y = -4x - 4$ and find the intercepts.

Answer

x -int is at $y=0$: $0 = -4x - 4$
 $(-1, 0)$
 $4 = -4x \Rightarrow x = \frac{4}{-4} = -1$

y -int is at $x=0$: $y = -4(0) - 4 = -4$
 $(0, -4)$



$Ax + By = C$ is the GENERAL FORM of a line.

Example

Graph the line $3x - 4y = 12$ on paper and on the calculator.

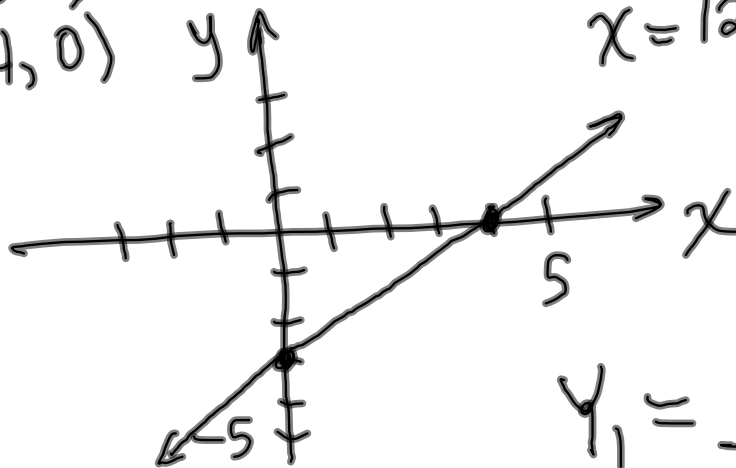
Answer

$$x=0 \Rightarrow 3(0) - 4y = 12 \Rightarrow -4y = 12 \Rightarrow y = 12 / -4 = -3$$

$(0, -3)$

$$y=0 \Rightarrow 3x - 4(0) = 12 \Rightarrow 3x = 12$$

$(4, 0)$ $x = 12 / 3 = 4$



$$y_1 = \underline{\hspace{2cm}}$$

$$3x - 4y = 12$$

$$-4y = 12 - 3x$$

$$y = (12 - 3x) / (-4)$$

Two lines are parallel if they have the same slope and different y-intercepts, $m_1 = m_2$ and $b_1 \neq b_2$

Two lines are perpendicular if the product of their slopes is -1,

$$m_1 \cdot m_2 = -1 \text{ or } m_1 = \frac{-1}{m_2}$$

Example

Given the line L_1 is $y = 2x + 4$,

(a) find a line parallel to L_1 that passes through the point $(4, 4)$

(b) find a line perpendicular to L_1 that passes through the point $(4, 4)$

Answer

$$m = 2$$

$$a) m_1 = 2 = m_2$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 2(x - 4) = 2x - 8$$

$$y + 4 = 2x - 8 + 4$$

$$y = 2x - 4$$

$$b) m_1 \cdot m_2 = -1 \text{ or } m_2 = \frac{-1}{m_1} = -\frac{1}{2}$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{2}(x - 4) = -\frac{1}{2}x + 2$$

$$y + 4 = -\frac{1}{2}x + 2 + 4$$

$$y = -\frac{1}{2}x + 6$$

APPLICATIONSExample ~~1990~~ 2010

In the ~~1990's~~ for wages less than the maximum taxable wage base, Social Security contributions by employees are 6.2% of the employee's wages.

a) Find a linear model that expresses the relationship between wages and Social Security contributions for employees earning less than the maximum (\$106,800 in 2010). $6.2\% = 6.2/100$

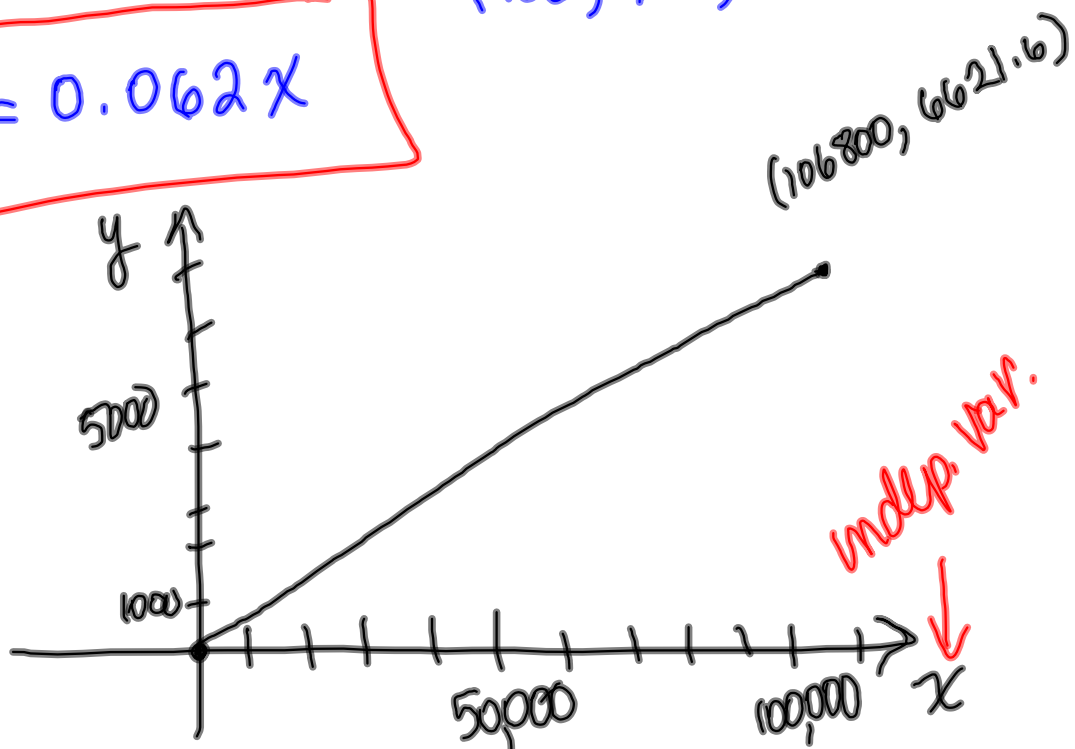
b) Graph this equation and find the social security contribution for an employee earning \$35,000 in wages in a year.

Answer

Let $x = \text{wages in } \$ \text{ and } x \leq 106,800$
 $y = \text{SS contrib. in } \$$

$(x, y) : (0, 0) \text{ and } (100, 100 \cdot 0.062)$
 $(100, 6.2)$

$$y = 0.062x$$

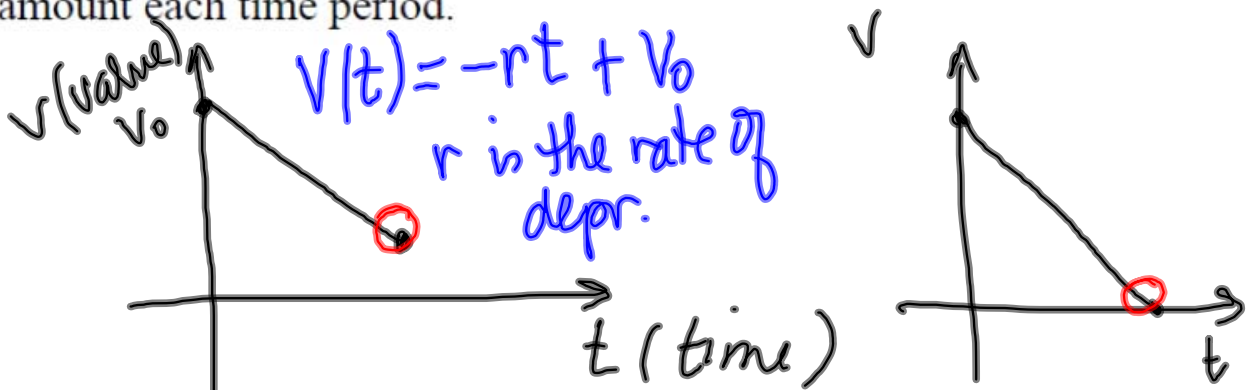


$$y = mx + b$$

LINEAR BUSINESS MODELS

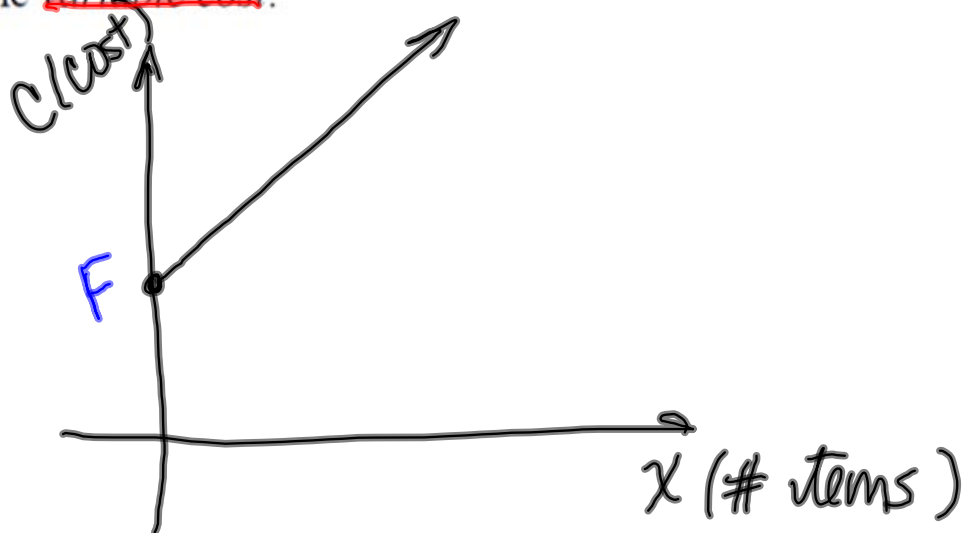
The rate of depr. of an item is given by the negative of the slope of the depr. line.

Depreciation: the value, V , of an item decreases linearly with time. The item has an initial value and then the value decreases by the same amount each time period.

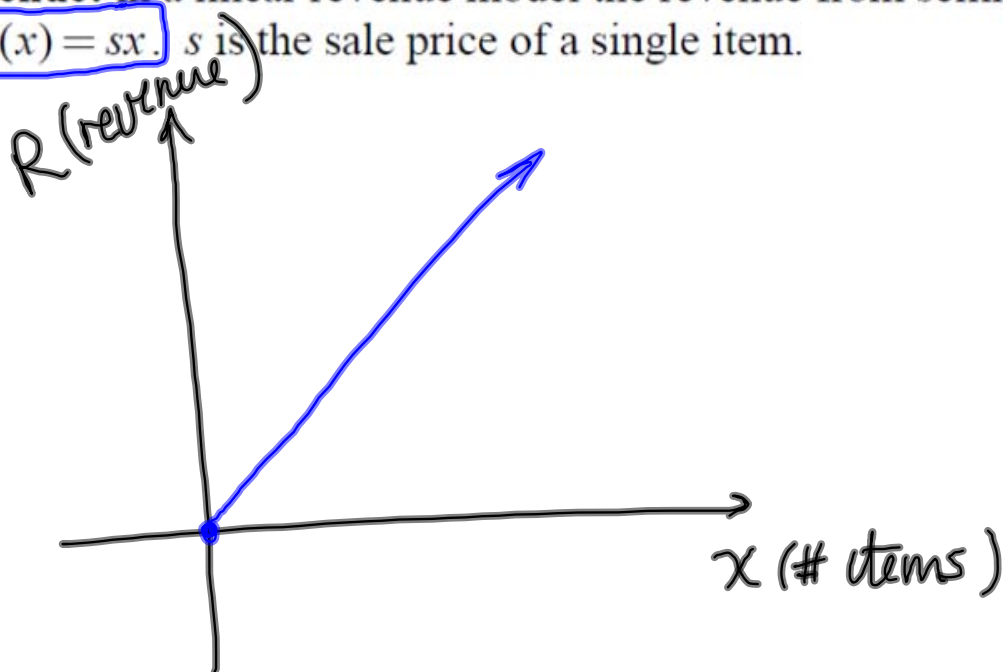


Scrap value: what the item is worth when you scrap it.

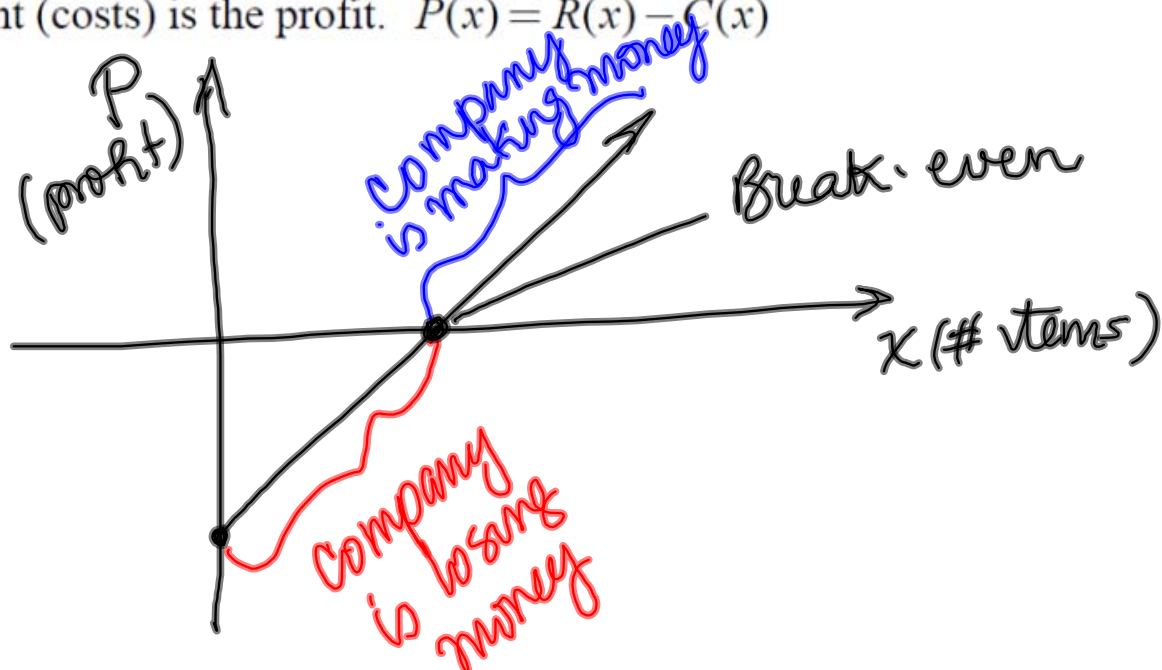
Cost: in a linear cost model the TOTAL cost to make x items is $C(x) = \underline{cx} + F$. F represents the *fixed costs*. These are the costs you have even if you make no items. c is the cost to make each unit, called the variable cost.



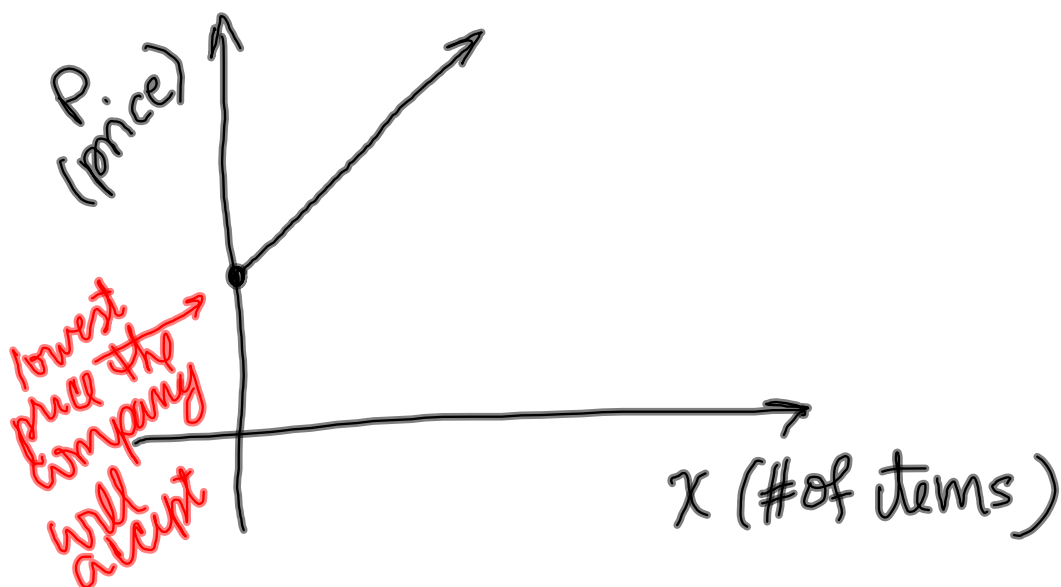
Revenue: in a linear revenue model the revenue from selling x items is $R(x) = sx$. s is the sale price of a single item.



Profit: the difference between the money in (revenue) and the money spent (costs) is the profit. $P(x) = R(x) - C(x)$

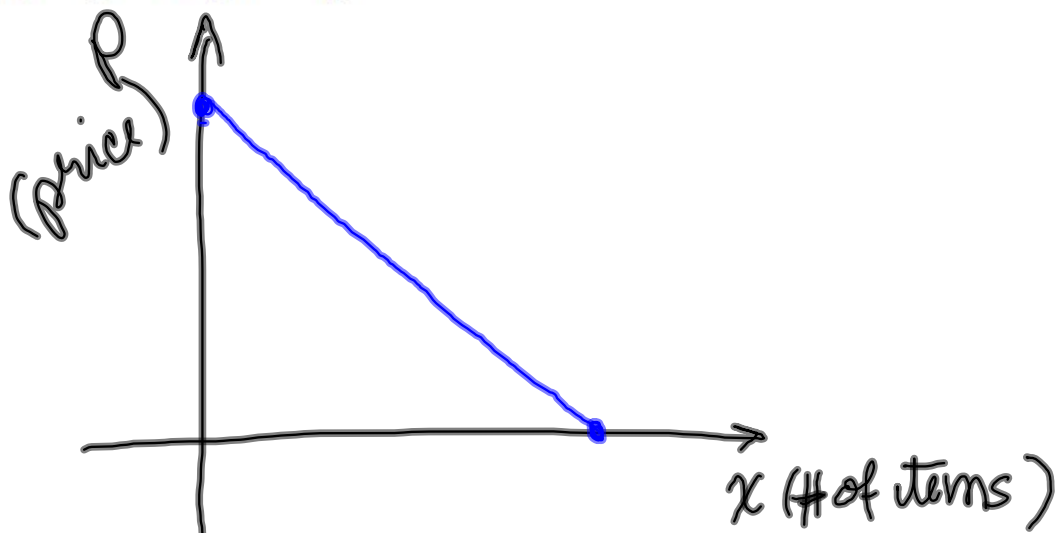


Supply: in a linear supply model the number of items, x , that a company will supply at a price p is given by $S(x) = p = m_s x + b_s$.



Demand: in a linear demand model the number of items, x , that consumers will purchase at a price p is given by

$$D(x) = p = m_D x + b_D.$$



DEPRECIATION*Example*

A car is purchased for \$18,000 and is kept for 7 years. At the end of 7 years the car is sold for \$4000. Find an equation that models the decrease in the value of the car over time. What is the car worth after 3 years?

alt: $(0, 18)$ and $(7, 4)$ with V in thousands of dollars

Answer

$$(t, v): (0, 18000) \text{ and } (7, 4000)$$

where t is the time in years
and V is the value in dollars

$$m = \frac{\Delta y}{\Delta x} = \frac{18000 - 4000}{0 - 7} = \frac{14000}{-7} = -2000$$

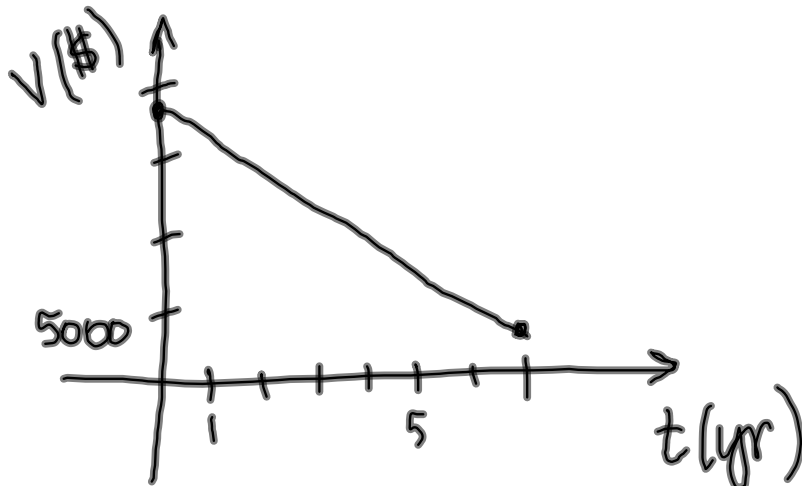
The rate of deprec is 2000 \$/yr

$$y - y_1 = m(x - x_1)$$

$$y - 18000 = -2000(x - 0) = -2000x$$

$$y = -2000x + 18000$$

$$\Rightarrow v(t) = -2000t + 18000$$



COST, REVENUE and PROFIT*Example*

Suppose a company manufactures baseball caps. In a day they can produce 100 caps for a total cost of \$600. If no caps are produced their costs are \$200 per day. The caps sell for \$8 each. Find the cost, revenue and profit equations. $m = \frac{600 - 200}{100 - 0} = \frac{400}{100} = 4$

Answer
 cost: $(x, C) = (100, 600)$ and $(0, 200)$

$$C(x) = 4x + 200$$

$x = \#$ of baseball caps produced
 $C =$ the total cost in \$

revenue:

$$R = 8x$$

$x = \#$ of baseball caps
 $R =$ revenue in \$

Profit

$$P = R - C$$

$$= 8x - (4x + 200)$$

$$P = 4x - 200$$

$x = \#$ of baseball caps
 $P =$ profit in \$

fixed and variable costs

SUPPLY AND DEMAND*Example*

A baker is willing to supply 16 jumbo cinnamon rolls to a café at a price of \$1.70 each. If she is offered \$1.50 for each roll, she will supply 4 fewer rolls to the café. At the café, customers will purchase no cinnamon rolls if the cost is \$7.20 each. However, if the price of a cinnamon roll is \$0.80, the café can sell 40 of these rolls.

Find the supply and demand equations for jumbo cinnamon rolls.

Supply
 $(x, p) = (16, 1.7) \text{ and } (16-4, 1.5)$
 $(12, 1.5)$

$$m = \frac{1.7 - 1.5}{16 - 12} = \frac{.2}{4} = 0.05$$

$$y - y_1 = m(x - x_1)$$

$$y - 1.5 = (.05)(x - 12)$$

$$y = 0.05x + .9$$

$$S(x) = p = 0.05x + .9$$

$x = \# \text{ of cin. rolls supplied}$
 $p = \text{price for each cin roll in } \$$

Demand

$$(x, p) = (0, 7.2) \text{ and } (40, .8)$$

$$m = \frac{7.2 - .8}{0 - 40} = \frac{6.4}{-40} = -.16$$

$$y - y_1 = m(x - x_1)$$

$$y - 7.2 = -.16(x - 0)$$

$$y = -.16x + 7.2$$

$$D(x) = p = -.16x + 7.20$$

$x = \# \text{ of cin. rolls demanded}$
 $p = \text{price per cin. roll in } \$$

THE INTERSECTION OF TWO LINES

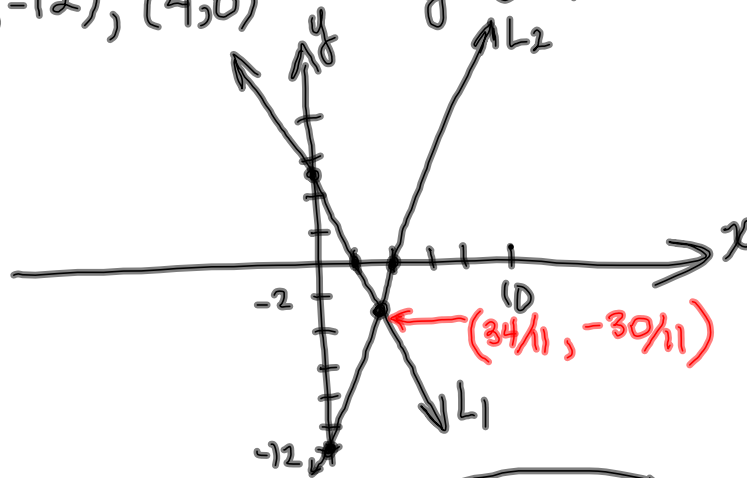
Find where the lines $10x + 4y = 20$ and $3x - y = 12$ intersect.

$$L_1: 10x + 4y = 20 \quad \begin{array}{l} x=0 \Rightarrow 4y=20 \Rightarrow y=5 \\ y=0 \Rightarrow 10x=20 \Rightarrow x=2 \end{array}$$

$(0,5), (2,0)$

$$L_2: 3x - y = 12 \quad \begin{array}{l} x=0 \Rightarrow -y=12 \Rightarrow y=-12 \\ y=0 \Rightarrow 3x=12 \Rightarrow x=4 \end{array}$$

$(0,-12), (4,0)$



$$L_1: 3x - y = 12 \Rightarrow y = 3x - 12$$

$$L_2: 10x + 4y = 20 \Rightarrow 10x + 4(3x - 12) = 20$$

$$10x + 12x - 48 = 20$$

$$22x = 68 \Rightarrow x = \frac{68}{22} = \frac{34}{11}$$

$$y = 3x - 12 = 3\left(\frac{34}{11}\right) - 12 = -\frac{30}{11}$$

$\left(\frac{34}{11}, -\frac{30}{11}\right)$

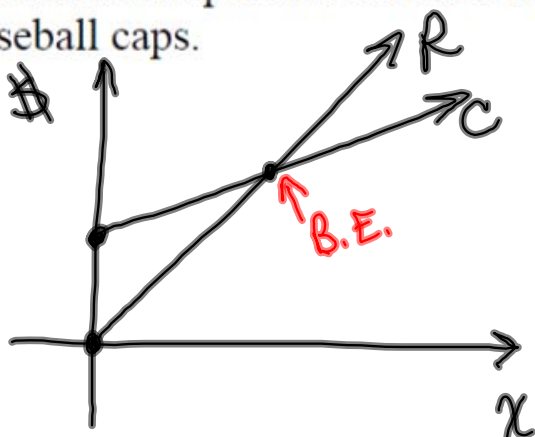
$$\frac{4y}{4} = \frac{20 - 10x}{4}$$

$$y = 5 - 2.5x$$

Break-even Point: This is where the cost to produce x items is the same as the revenue brought in from selling these x items. This occurs when $R(x) = C(x)$.

Example

Find and interpret the break-even point for making and selling baseball caps.



$$R(x) = C(x)$$

$$8x = 4x + 200$$

$$-4x \quad -4x$$

$$4x = 200 \Rightarrow x = \frac{200}{4} = 50$$

$$R(50) = 8(50) = 400$$

$$C(50) = 4(50) + 200 = 400$$

B.E. at $(50, 400)$ which means that when 50 caps are made and sold, the profit is zero, and the revenue and cost for these caps is \$400.

Equilibrium Point: This is the price p that the consumer and producer are willing to pay/accept for x items. This occurs when $S(x) = D(x)$

Example

Find and interpret the equilibrium point for the supply and demand for jumbo cinnamon rolls.

$$S(x) = p = 0.05x + .9$$

$$D(x) = p = -0.16x + 7.2$$

$$S(x) = D(x)$$

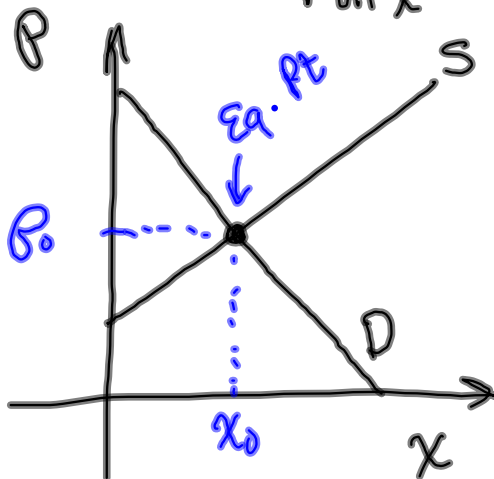
$$0.05x + .9 = -0.16x + 7.2$$

$$+ .16x \qquad \qquad \qquad + .16x$$

$$.21x + .9 = 7.2$$

$$- .9 \qquad \qquad \qquad - .9$$

$$.21x = 6.3 \Rightarrow x = \frac{6.3}{.21} = 30$$



$$S(30) = 0.05(30) + .9$$

$$= 2.4$$

$$D(30) = -.16(30) + 7.2$$

$$= 2.4$$

Eq. pt is $(30, 2.4)$ which means that 30 cinnamon rolls are supplied and demanded at a price of \$2.40 each.