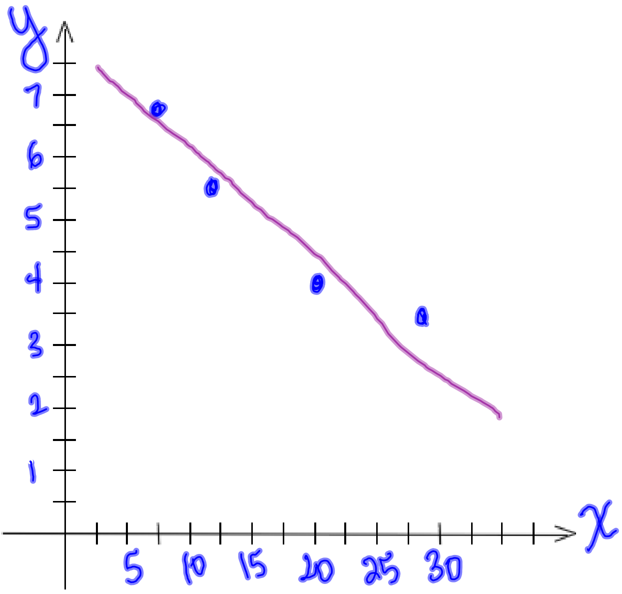
WEEK 3 REVIEW –
Linear Regression and Systems of Linear Equations

Example
The table below shows x, the number of cartons of blueberries, that a fruit stand can sell at different prices y in dollars.

x	8	11	20	28
y	6.9	5.5	4	3.35

(a) Show this data in a scatter diagram (plot)



(b) Use linear regression to find the best-fitting line for the price of blueberries. To 4 decimal places,

$$y = -0.1674x + 7.7419$$

Press the STAT button and then press ENTER to edit your lists	Enter the values for *x* in list L1 and *y* in list L2	Press 2ND and QUIT to return to the homescreen. Press STAT and right arrow to CALC	Choose 4:LineReg(ax+b) and press ENTER:
EDIT CALC TESTS 1:Edit… 2:SortA(3:SortD(4:ClrList 5:SetUpEditor	L1 L2 L3 2 8 6.9 ------ 11 5.5 20 4 28 3.35 ------ ------ L2(1)=6.9	EDIT CALC TESTS 1:1-Var Stats 2:2-Var Stats 3:Med-Med 4:LinReg(ax+b) 5:QuadReg 6:CubicReg 7↓QuartReg	LinReg(ax+b)

Use L1 for x and L2 for y	Press ENTER to get the best-fitting line:	If r and r^2 are not displayed, go to CATALOG (2ND and the 0 button) and choose DiagnosticON.	To graph the data and best-fitting line, press 2ND and Y= to access the STATPLOT
LinReg(ax+b) L1, L2	LinReg y=ax+b a=-.1674265451 b=7.74189463 r²=.9140905782 r=-.9560808429	CATALOG Degree DelVar DependAsk DependAuto det(DiagnosticOff ▸DiagnosticOn	STAT PLOTS 1:Plot1…On L1 L2 2:Plot2…Off L1 L2 3:Plot3…Off L1 L2 4↓PlotsOff

Press ENTER to set-up Plot1:	To enter the best-fitting line, press Y= and then the VARS button:	Choose 5: Statistics and right arrow twice to EQ:	Press ENTER to paste RegEQ into Y1=:
Plot1 Plot2 Plot3 On Off Type: Xlist:L1 Ylist:L2 Mark: ▫ + ·	VARS Y-VARS 1:Window… 2:Zoom… 3:GDB… 4:Picture… 5:Statistics… 6:Table… 7:String…	XY Σ EQ TEST PTS 1:RegEQ 2:a 3:b 4:c 5:d 6:e 7↓r	Plot1 Plot2 Plot3 \Y1=-.1674265450 8612X+7.74189463 01925 \Y2= \Y3= \Y4= \Y5=

(c) What is the selling price of blueberries when 35 cartons are sold?

$x = 35$

$y = -0.1674(x) + 7.7419$

$= 1.8829 \Rightarrow$ price is \$1.88

on calc, $y = 1.8819656\ldots$
so watch the rounding

(d) If the fruit stand charges \$5.00 for a carton of blueberries, use the best-fitting line to estimate how many cartons will be sold.

$5 = -0.1674(x) + 7.7419$

$x = 16.37933\ldots$

$\Rightarrow$ 16 cartons of blueberries will be sold.

on calc, intersect $y = 5 \Rightarrow x = 16.3767\ldots$
still 16 cartons

(e) If the data in the table represented the number of cartons of blueberries in thousands sold by a large grocery chain, estimate how many cartons will be sold at a price of \$3.50 per carton.

$3.50 = -0.1674(x) + 7.7419$

$\Rightarrow x = 25.3399$ th. of cartons

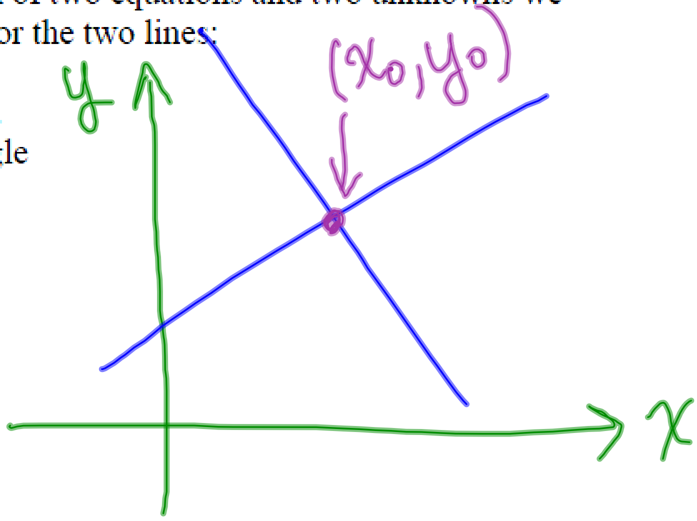
$1000x = 25{,}340$ cartons

on calc, 25,336 cartons

SYSTEMS OF LINEAR EQUATIONS

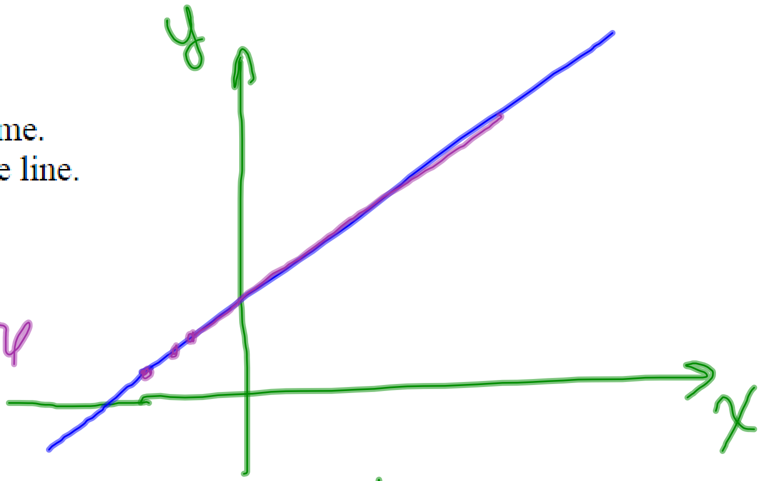
When you have a system of two equations and two unknowns we have three possibilities for the two lines:

The two lines intersect.
The solution is the single point of intersection.

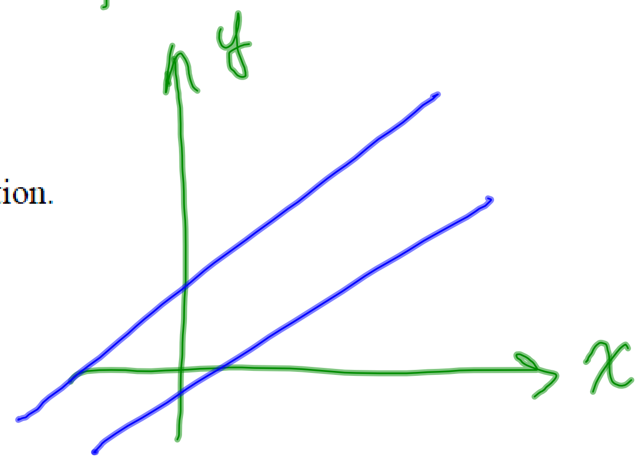


The two lines are the same.
The solution is the entire line.

parametric
usually there are infinite solns.



The two lines are parallel.
No intersection therefore no solution.



Soln is $(x,y) =$ ______

Example:
Solve the following systems of linear equations.

(a) $$\begin{aligned} x+2y&=12 \\ 2x+3y&=19 \end{aligned}$$

$x = 12-2y = 12-2(5)$

$x = 2$

$2(12-2y)+3y = 19$

$24-4y+3y = 19$

$-y = -5$

$y = 5$

$(x,y) = (2,5)$

(b) $\begin{array}{l} 2x - 4y = 8 \\ -x + 2y = 4 \end{array}$ $\Rightarrow -4y = 8 - 2x \rightarrow y = -2 + \frac{1}{2}x$

$$-x + 2\left(-2 + \tfrac{1}{2}x\right) = 4$$

$$\cancel{-x} - 4 + \cancel{x} = 4$$

$$-4 = 4 \quad \text{NOT TRUE}$$

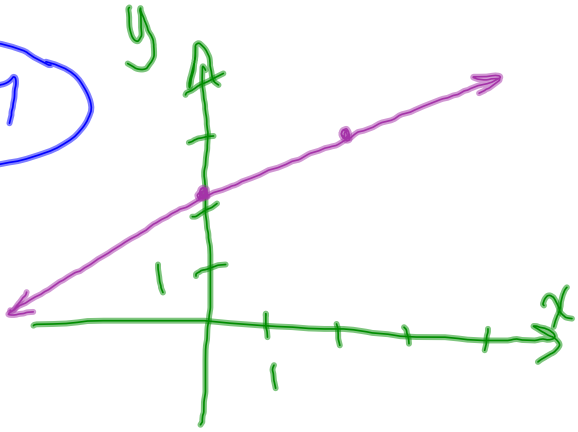
slope-intercept form: $y = \frac{1}{2}x - 2$

$2y = x + 4 \rightarrow y = \frac{1}{2}x + 2$

NO SOLUTION

$(x, y) =$ no solution

(c) $\begin{array}{l} -x+3y=7 \\ 2x-6y=-14 \end{array}$ $\longrightarrow$ $x = 3y - 7$



$2(3y-7) - 6y = -14$

$\cancel{6y} - 14 - \cancel{6y} = -14$

$-14 = -14$

$0 = 0 \Rightarrow$ TRUE

$y = (1/3)x + 7/3$

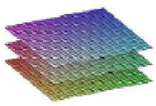
$\boxed{(x,y) = (3y-7,\, y) \quad y \text{ is any } \#}$ or $(x,\, 1/3\, x + 7/3)$

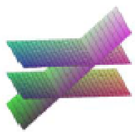
$(x,y) = (3t-7,\, t)$ t is any #

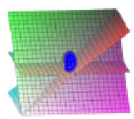
Number of Solutions Theorem

If the number of equations in a system of linear equations is equal to or greater than the number of variables, the system may have

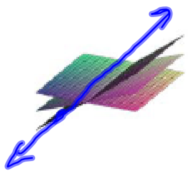
- No solution
- Exactly one solution
- A parametric solution



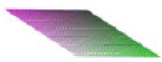




(x_0, y_0, z_0)

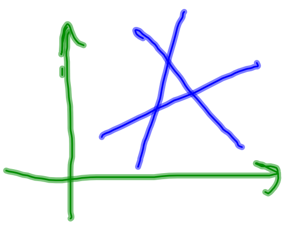


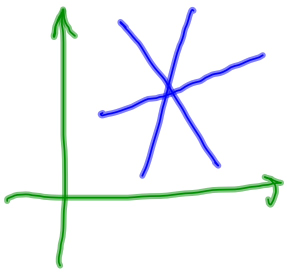
(1 par)

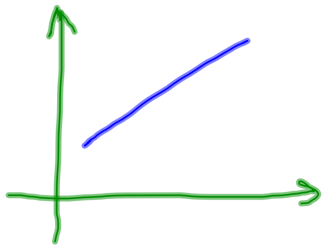


(2 parameters)

(x, y, z) and 3 planes

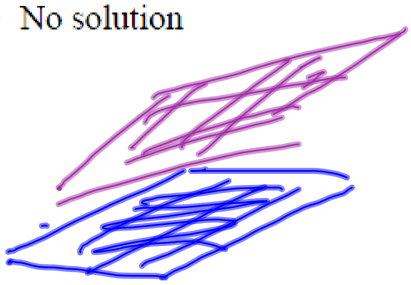




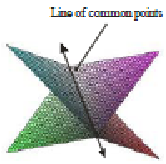


If the number of equations in a system of linear equation is less than the number of variables, then the system may have

- No solution



- A parametric solution



Set it up

WORD PROBLEMS

Example:

Jane invests \$10,000 in three ways. With one part she buys mutual funds with a return of 6.5% per year. The second part (which is twice as large as the 1st part) is used to buy government bonds that pay 6% per year. The rest is put into a savings account paying 5% per year. In the 1st year her average return was 6.05%.. How much did she invest in each way?

x = amount of money in \$ invested in the mutual fund.

y = " " " " " " " " gov. bonds

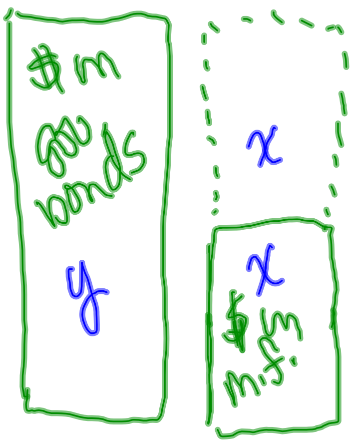
z = " " " " " " " " savings acct.

$$x + y + z = 10000$$ (total money invested in \$)

$$y = 2x$$ (ratio of gov. bonds to mutual funds)

$$0.065x + 0.06y + 0.05z = 10000(0.0605) = 605$$

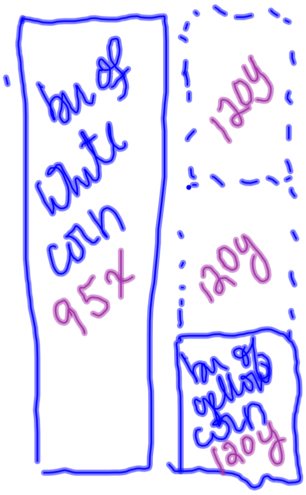
(interest earned from all 3 investments)



Example:
Farmer Blue has 100 acres available to plant white and yellow corn. Each acre of white corn will yield 95 bushels of corn and each acre of yellow corn will yield 120 bushels of corn. He wants to have ~~at least~~ three times as many bushels of white corn than he does of yellow corn. How many acres of each type of corn should Farmer Blue plant?

x = # of acres of white corn
y = # of acres of yellow corn

$x + y = 100$ (total available acres)
$95x = 3(120y)$ (ratio of white to yellow bu)



GAUSS-JORDAN

Example

Solve the following system of linear equations:

$$\begin{aligned} -3x-6y+6z&=-3 \\ 2x+7y+2z&=-1 \\ -x-6y-7z&=3 \end{aligned} \longleftrightarrow \begin{aligned} x&=a \\ y&=b \\ z&=c \end{aligned}$$

1. Any two ~~equations~~ rows may be interchanged.
2. An ~~equation~~ row may be multiplied by a non-zero constant.
3. A multiple of one ~~equation~~ row may be added to another ~~equation~~ row.

Augmented Matrix

$$\begin{array}{c} \begin{array}{cccccc} x & y & z & = & \# \end{array} \\ \left[\begin{array}{ccc|c} -3 & -6 & 6 & -3 \\ 2 & 7 & 2 & -1 \\ -1 & -6 & -7 & 3 \end{array}\right] \end{array}$$

RREF

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right]$$

A matrix is in Reduced-Row Echelon Form if (RREF)

1. Each row consisting entirely of zeros lies below any row having non-zero entries.
2. The 1st non-zero entry in any row is a 1 (called a leading 1)
3. In any two successive (non-zero) rows the leading 1 in the lower row lies to the right of the leading 1 in the upper row.
4. If a column contains a leading 1, the rest of the column is 0.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & \\ 0 & 0 & 1 & \end{array}\right]$$

IMPORTANT!

- Only consider entries to the LEFT of the vertical line when Appling the definition of RREF.
- If a matrix is in RREF form, it may have one solution, no solution or a parametric solution.

Pivot → make the element a "1" and the rest of the col. zeros.

$$-3x-6y+6z=-3$$
$$2x+7y+2z=-1$$
$$-x-6y-7z=3$$

Pivot on Row 1 col 1
then Pivot on Row 2 col 2
finally pivot on Row 3 col 3

make the "1" first. Do this by multip. by the recip

$$\left[\begin{array}{ccc|c} -3 & -6 & 6 & -3 \\ 2 & 7 & 2 & -1 \\ -1 & -6 & -7 & 3 \end{array}\right] \xrightarrow{-\frac{1}{3}R_1 \to R_1} \left[\begin{array}{ccc|c} (-3)(-\frac{1}{3}) & (-6)(-\frac{1}{3}) & 6(-\frac{1}{3}) & (-3)(-\frac{1}{3}) \\ 2 & 7 & 2 & -1 \\ -1 & -6 & -7 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 2 & 7 & 2 & -1 \\ -1 & -6 & -7 & 3 \end{array}\right] *(-2)$$

Since $-2+2=0$ multiply R_1 by -2 and add to row 2

$$\xrightarrow{-2R_1+R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 2+(-2)(1) & 7+(-2)(2) & 2+(-2)(-2) & -1+(-2)(1) \\ -1 & -6 & -7 & 3 \end{array}\right]$$

$$\to *1 \left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 3 & 6 & -3 \\ -1 & -6 & -7 & 3 \end{array}\right]$$

Since $-1+1=0$ multiply row 1 by 1 and add to row 3

$1*R_1+R_3$: $\frac{(1)(1)}{0}$ $\frac{(2)(1)}{-4}$ $\frac{(-2)(1)}{-9}$ $\frac{(1)(1)}{4}$

$$\xrightarrow{1*R_1+R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 3 & 6 & -3 \\ 0 & -4 & -9 & 4 \end{array}\right]$$

done pivoting on Row 1 Col 1

next, pivot row 2, col 2

Pivot on Row 2 Col 2

$$\left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 3 & 6 & -3 \\ 0 & -4 & -9 & 4 \end{array}\right] \xrightarrow{\frac{1}{3}R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & -4 & -9 & 4 \end{array}\right] \times -2$$

be a zero

$$\xrightarrow{-2R_2 + R_1 \to R_1} \begin{array}{cccc} +(-2)(0) & +(1)(-2) & +(2)(-2) & +(-1)(-2) \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -2 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & -4 & -9 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & -6 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & -4 & -9 & 4 \end{array}\right]$$

be a zero

$$\xrightarrow{4R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -6 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & -4 & -9 & 4 \end{array}\right] \begin{array}{cccc} +4(0) & +(4)(1) & +(4)(2) & +(4)(-1) \end{array}$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & -6 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -1 & 0 \end{array}\right]$$

Pivot on Row 3 Col 3

$$\left[\begin{array}{ccc|c} 1 & 0 & -6 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & -1 & 0 \end{array}\right] \xrightarrow{-1R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & -6 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \leftarrow *6$$

$$\xrightarrow{6R_3 + R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \leftarrow *-2$$

$$\begin{array}{cccc} x & y & z & = \# \end{array}$$

$$\xrightarrow{-2R_3 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} x = 3 \\ y = -1 \\ z = 0 \end{array}$$

Example
Solve the following systems of linear equations:

$$\text{(a)}\quad \begin{aligned} 2x+y-z&=0\\ 3x-y+2z&=1\\ x-2y+3z&=2 \end{aligned}$$

$$\begin{array}{cccc} x & y & z & = \# \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & .2 & 0 \\ 0 & 1 & -1.4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

RREF?

$x + .2z = 0$

$y - 1.4z = 0$

$0 = 1 \Rightarrow$ NO SOLN

(b) $2x + y - 4z = 10$
$x + 2y + z = 5$
$x + y - z = 5$

$\xrightarrow{\text{RREF}}$ $\begin{array}{cccc} x & y & \cancel{z} \; t & \# \\ \end{array}$ $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 5 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x - 3z = 5$
$y + 2z = 0$
$0 = 0$

Parametric

$(x, y, z) = (5+3t, -2t, t)$, t is any #

x, y are basic variables (col has a leading 1)
z is a non-basic variable $\rightarrow$ a parameter
$\Rightarrow z = t$
$x - 3t = 5 \rightarrow 5+3t$
$y + 2t = 0 \rightarrow y = -2t$

part. solns $\rightarrow$ pick values for t
$t = 0 \Rightarrow (5+3(0), (-2)(0), (0)) = (5, 0, 0)$
$t = 1 \Rightarrow (8, -2, 1)$

Example

A zoo is looking to acquire some lions, tigers and bears. The zoo has 2800 square feet of space available and \$850 for transportation costs. A lion needs 200 square feet of space and costs \$50 to transport. A tiger needs 400 square feet of space and costs \$150 to transport. A bear needs 400 square feet of space and costs \$50 to transport. How many lions, tigers and bears can the zoo get?

x = # of lions
y = # of tigers
z = # of bears

$200x + 400y + 400z = 2800$ (sq ft available)
$50x + 150y + 50z = 850$ (\$ avail for transp)

$$\left[\begin{array}{ccc|c} 200 & 400 & 400 & 2800 \\ 50 & 150 & 50 & 850 \end{array}\right] \xrightarrow{\text{rref}} \begin{array}{cccc} x & y & \cancel{z}\ t & = \# \\ \left[\begin{array}{ccc|c} 1 & 0 & 4 & 8 \\ 0 & 1 & -1 & 3 \end{array}\right] \end{array}$$

$$\begin{aligned} x + 4t &= 8 \\ y - t &= 3 \end{aligned} \Rightarrow (x, y, z) = (8 - 4t, t + 3, t)$$

t is # of bears

$t = 0: (8 - 4(0), 0 + 3, 0) \to (8, 3, 0)$
Buy 8 lions, 3 tigers and 0 bears

$t = 1: (8 - 4(1), 1 + 3, 1) \to (4, 4, 1)$
Buy 4 lions, 4 tiger and 1 bear

$t = 2: (8 - 4(2), 2 + 3, 2) \to (0, 5, 2)$
Buy 0 lions, 5 tigers and 2 bears