

WEEK 13 REVIEW - Finance Part 1

SIMPLE INTEREST

Simple interest is interest computed as a percentage of the principal

Simple interest earned:  $I = Prt$

$I$  = interest earned

$P$  = principal

$r$  = interest rate (decimal form) per year

$t$  = time period (in years)

Accumulated amount (simple interest):  $A = P + Prt = P(1 + rt)$

Example

If a bank loans \$678 to an individual for 2½ years at 5.75% simple interest, what will be the amount repaid on the loan?

$I = 678 * 0.0575 * 2.5 = 974.625$

repay  $678 + 974.6 = 1652.625$

Example

If a \$1000 deposit grows in value to \$1024 after 8 months, what is the simple interest rate that is earned?

$1024 = 1000(1 + r * 8/12)$

$1024 = 1000 + r * 666.67$

$24 = 666.67 r$

$r = 0.036$

$3.6\%$

①

COMPOUND INTEREST

Compound interest - interest that is computed periodically as a percentage of the sum of the principal and the interest already accrued

Example

If a person borrows \$4000 for two years at an interest rate of 6.49% compounded annually, what is the amount that must be repaid on the loan at the end of the two years?

end of 1st year,  $A = 4000(1 + 0.0649 \cdot 1) = 4259.6$

end of 2nd year  $A = 4259.6(1 + 0.0649 \cdot 1) = 4536.05$

Example

A sum of \$100 is invested at 10% annual interest compounded quarterly. How much is in the account after 1 year.

end of 1st Q,  $A_1 = 100(1 + 0.025) = 102.5$

2nd Q,  $A_2 = 102.5(1 + 0.025) = 105.0625$

3rd Q,  $A_3 = 105.0625(1 + 0.025) = 107.689...$

4th Q,  $A_4 = 107.689(1 + 0.025) = 110.3812...$

$110.38$

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Using the TVM Solver Application

Locating TVM Solver: APPS->Finance...->TVM Solver...

Parameters in TVM Solver:

- **N**: total compounding periods;
- **N** = (# of compounding periods per year)\*(# of years)
- **I%**: interest rate (percent form)
- **PV**: present value (principal amount)
- **PMT**: payment amount
- **FV**: future value (accumulated amount)
- **P/Y**: number of payments per year
- **C/Y**: number of compounding periods per year
- **PMT: END BEGIN**: indicates whether payments are made at beginning or end of compounding period

If interest is compounded	Then P/Y = C/Y =
Annually	1
Semiannually	2
Quarterly	4
Monthly	12
Weekly	52
Daily	365

Example

A sum of \$10,000 is invested at 5% annual interest for 1 year. Determine the interest earned when the account is compounded

- a) Annually  
 $N = 1 \times 1 = 1$      $PV = 10000$      $FV = ?$      $I\% = 5$      $PMT = 0$      $P/Y = 1$   
 $FV = 10500$      $\$500$
- b) Semi-annually  
 $N = 2 \times 1 = 2$      $FV = 10506.25$      $\$506.25$   
 $P/Y = 2$
- c) Quarterly  
 $N = 4 \times 1 = 4$      $FV = 10509.45337$      $\$509.45$   
 $P/Y = 4$
- d) Monthly  
 $N = 12 \times 1 = 12$      $FV = 10511.61898$      $\$511.62$   
 $P/Y = 12$
- e) Weekly  
 $N = 52$      $FV = 10512.45842$      $\$512.46$   
 $P/Y = 52$
- f) Daily  
 $N = 365$      $FV = 10512.67496$      $\$512.67$   
 $P/Y = 365$
- g) Hourly  
 $N = 365 \times 24 = 8760$      $FV = 10512.70946$      $\$512.71$   
 $P/Y = 8760$

Continuous compound interest

Accumulated amount (continuous compound):  $A = Pe^{rt}$

$A$  = accumulated amount

$P$  = principal

$r$  = interest rate (decimal form)

$t$  = time (years)

\*Note:  $e$  is the natural number.  $e \approx 2.718 \dots$

Example

A sum of \$10,000 is invested at 5% annual interest for 1 year. Determine the interest earned when the account is compounded continuously for 1 year.

$$A = 10000 \cdot e^{0.05 \cdot 1} = 10512.71096 \dots$$

Effective interest rate

The effective interest rate — is the corresponding annually compounded interest rate that produces the same interest as a given annual rate in one year

\*Computing effective rate using TI-83/84:

APPS → Finance → Eff

Eff( interest%, number of compounding periods per year)

Example: The Bank of the North offers a savings account that pays 2.6% interest, compounded daily. The Bank of the East offers a similar savings account that pays 2.55%, compounded ~~daily~~ ~~monthly~~. Compute the effective rate for each bank, and determine which bank offers the better deal for an investor.

$$\text{Eff}(2.6, 4) = 2.6255\%$$

$$\text{Eff}(2.55, 365) = 2.6135\%$$

Example: Joey purchases 100 shares of a particular stock for a total of \$20,000 in 2006. He sold the shares in 2008 for \$22,222. Determine the effective annual rate of return on the investment, rounded to two decimal places.

$$N = 2 \quad PMT = 0$$

$$I = ? \quad PV = 22222$$

$$PV = 20000 \quad FV = 1$$

sign change!

$$\rightarrow I = 5.1409\%$$

$$\text{In } 2009, FV = 141500 \rightarrow I = -10.1649\%$$

OTHER FINANCE FNS ...  
dbd (DDMM.YY, DDMM.YY)

Example

What is the amount repaid on a loan of \$600, if the loan charges 8.6% compounded continuously for 44 months?

$$A = 600 e^{0.086 \times 44/12} = 822.43$$

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Example

The Bank of the West offers a savings account that pays 3.15% compounded weekly. If Dorothy invests \$400 in this type of savings account for twenty years, what will be the amount in her account at the end of twenty years?

$$N = 20 \times 52 = 1040 \quad PMT = 0$$

$$I = 3.15 \quad FV = ?$$

$$PV = 400 \quad P/Y = 52$$

$$FV = \boxed{\$750.90}$$

Time to double?  $FV = -800, N = ?$   
 $N = 1144.6 \text{ weeks}$   
 $\approx 22 \text{ yrs}$

Example

Webster borrows \$250 at an interest rate of 12% compounded monthly. At the end of two years, what will be the amount to be paid on the loan?

$$N = 2 \times 12 = 24 \quad PMT = 0$$

$$I = 12 \quad FV = ?$$

$$PV = 250 \quad P/Y = 12$$

$$FV = \$317.43$$

Example

Suppose you invest \$1000 in a savings account toward the purchase of an Aggie ring with a price of \$1200. If interest earned by the account is 4% compounded daily, how long will it take your investment to grow to \$1200?

$$N = ? \quad FV = -1200$$

$$I = 4 \quad P/Y = 365$$

$$PV = 1000 \quad \rightarrow N = 1664 \text{ days}$$

$$PMT = 0 \quad \approx 4.6 \text{ yrs}$$

@ 5%,  $N = 1331 \text{ days} \approx 3.6 \text{ yrs}$   
 @ 6%,  $N = 1109 \text{ days} \approx 3 \text{ yrs}$

Example

Sandy is planning a vacation to Japan in two years. If the estimated cost of her trip is \$2700, how much should she deposit now in an account in order to have the money in three years, if the account pays 6% compounded semiannually?

$$N = 2 \times 2 = 4 \quad PMT = 0$$

$$I = 6 \quad FV = 2700$$

$$PV = ? \quad P/Y = 2$$

$$PV = \$2398.92$$

(earn \$301.08 in interest)

ANNUITIES

An annuity is an account to which regular payments are made. An annuity that is certain and simple has the following properties:

1. The payments are made at fixed time intervals
2. The periodic payments are of equal size
3. The payments are made at the end of the interval
4. The interest is paid at the end of the interval

Many loans and savings plans are certain and simple annuities

Example

Suppose you want to save for an Aggie ring with a price of \$1200 by making regular monthly deposits. If you find an account that pays 4% compounded monthly for 36 months, how much needs to be deposited each month? How much interest is earned?

$N = 36$      $PMT = ?$      $PMT = \$31.43$   
 $I = 4$          $FV = 1200$          $SAVE\ 36 \times 31.43$   
 $PV = 0$          $P/Y = 12$              $= \$1131.48$

earn 1200 - 1131.48 = \$68.52

Example

You are saving for a pair of diamond earrings. You put away \$10 every week into an account that pays 8% compounded weekly. How long until you have \$800? How much interest is earned?

$N = ?$      $PMT = 10$      $N = 85.4$   
 $I = 8$          $FV = 800$   
 $PV = 0$          $P/Y = 52$

$N = 75.5\ weeks \rightarrow 76$   
 $saved\ 10 \times 76 = \$760$   
 $\$40\ in\ interest$

Example

To save for retirement you make annual payments of \$3000 to an account that pays 9% annual interest compounded annually. How much will you have after saving for 10 years? For 20 years? For 30 years? How much interest is earned in each case?

$N = 10\ (20, 30)$      $FV(10) = \$45579$   
 $I = 9$                  $-10(3000) = \$15,579\ int.$   
 $PV = 0$   
 $PMT = 3000$          $FV(20) = \$153,480$   
 $FV = ?\ (?, ?, ?)$      $-20(3000) = \$93,480$   
 $P/Y = 1$               $FV(30) = \$408,923$   
                               $-30(3000) = \$318,923$

Example

You find that you can afford a monthly mortgage payment of \$1600. You find a loan that charges 6% annual interest compounded on the outstanding balance each month for 25 years. How large of a loan can you get? How much interest will be paid in all?

$N = 12 \times 25 = 300$      $PV = 248,331$   
 $I = 6$                  $pay\ back\ 300(1600) = 480,000$   
 $PV = ?$               $interest = \$231,669$   
 $PMT = 1600$   
 $FV = 0, P/Y = 12$

Example

A lottery prize is \$20,000,000 which is paid in 20 annual payments of \$1,000,000 each. If the winner is given \$1,000,000 now, how much needs to be deposited in an account paying 8.5% annual interest compounded annually if the winner is to be paid \$1,000,000 per year at the end of the year for the next 19 years?

$N = 19$      $PMT = 1,000,000$   
 $I = 8.5$          $FV = 0$   
 $PV = ?$          $P/Y = 1$   
  
 $PV = 9,267,720$   
 $+ 1,000,000\ (1^{st}\ PMT)$   
 $10,267,720\ (\approx 1/2\ price\ "value")$