AMORTIZATION

Example: A credit card charges 24% annual interest compounded monthly on the unpaid balance. You owe $4000 on this credit card. To pay it off, you stop using it and make monthly payments of $100.
(a) How long until the card is paid off?
(b) How much was paid in interest charges?
(c) How much of the first payment was interest?
(d) How much of the second payment was interest?
(e) How much is still owed on the credit card after two years of making payments?

Example: A house costs $180,000. You make a down payment of $20,000 and finance the remainder for 20 years at 5.1% annual interest compounded monthly on the unpaid balance.
(a) How large are the monthly payments?
(b) How much interest is paid in all?
(c) Fill in the blanks on the amortization table below:

<table>
<thead>
<tr>
<th>end of period</th>
<th>remaining payments</th>
<th>payment</th>
<th>interest paid</th>
<th>towards principal</th>
<th>outstanding balance</th>
<th>equity</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>360</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
<td>12</td>
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<td>60</td>
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<tr>
<td>120</td>
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<tr>
<td>180</td>
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</tbody>
</table>
**MARKOV CHAINS**

*Example*
Bob buys a cup of coffee or tea every day. If he buys a cup of coffee, there is a 30% chance he will buy a cup of coffee the next day and a 70% chance he will buy a cup of tea. If he buys a cup of tea, there is a 50% chance that he will buy a cup of tea the next day and a 50% chance he will buy a cup of coffee.

(a) Is this a Markov process?
(b) Find the transition matrix.
(c) On his first day back from vacation, Bob buys a cup of coffee. What is the probability that he buys a cup of coffee 3 days later?
(d) What are the long term (steady state) probabilities that Bob buys a cup of coffee or tea?

*Example*
In a certain city elections are held every two years for mayor. There are three political parties in this city, A, B and C. If the current mayor is from the A party, there is a 20% chance that the next mayor will be from the A party, 40% from the B party and 40% from the C party. If the current mayor is from the B party, there is a 50% chance the next mayor will be from the A party and a 50% chance that the mayor will be from the C party. If the current mayor is from the C party, there is a 60% chance the next mayor will be from the C party, a 10% chance the next mayor will be from the A party and a 30% chance the next mayor will be from the C party.

(a) Is this a Markov process?
(b) Find the transition matrix.
(c) On the city’s first election there is an equal chance that the mayor comes from parties A, B and C. What is the probability that the mayor is from party A in 8 years?
(d) What is the long term (steady state) distribution of the mayor’s political party?

*Example*
There are three brands of cell phones given to employees of a company. Each year the employee can choose phone brand X, phone brand Y or phone brand Z. If an employee has a brand X phone, he will choose a brand X phone again the next year. If an employee has a brand Y phone, there is an equal chance that he will choose brand X, Y or Z the next year. If a person has a brand Z phone, he will choose a brand X phone 50% of the time, a brand Y phone 25% of the time and a brand Z phone 25% of the time.

(a) Is this a Markov process?
(b) Find the transition matrix.
(c) Initially all employees are given brand Z phones. What is the distribution of phone ownership in 2 years?
(d) What is the long term (steady state) distribution of phone ownership?

*Example*
Classify the following matrices as a regular transition matrix, not a regular transition matrix, or not a transition matrix.

(a) \[
\begin{bmatrix}
0 & 0.4 \\
1 & 0.6 \\
\end{bmatrix}
\]

(b) \[
\begin{bmatrix}
0.5 & 0.8 \\
0.5 & 0.6 \\
\end{bmatrix}
\]

(c) \[
\begin{bmatrix}
0.75 & 0 \\
0.25 & 1 \\
\end{bmatrix}
\]