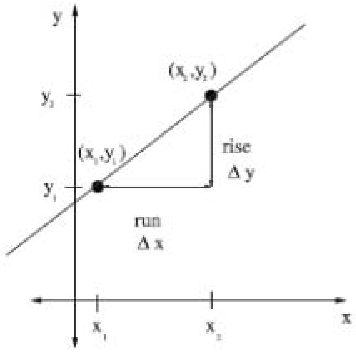
http://www.math.tamu.edu/~epstein/FiniteMath/

WEEK 1 REVIEW – Lines and Linear Models

SLOPE

A VERTICAL line has NO SLOPE. All other lines have

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Example
Find the slope of the line passing through the points (–2, 4) and (0, –4)

Answer
Let one pair of points be (x_1, y_1) and the other (x_2, y_2). Then

EQUATIONS OF LINES

The formula for the slope of a line can be rearranged to give us the equation for a line.

$$m = \frac{y - y_1}{x - x_1} \rightarrow \boxed{y - y_1 = m(x - x_1)}$$

This is called the POINT-SLOPE form of a line. If you know a point, (x_1, y_1) that lies on the line and you know the slope, m, of the line, then you can find the equation of the line.

Example
What is the equation of the line passing through the points (–2, 4) and (0, –4)?

Answer
$m = -4$ (previous example) Let $(x_1, y_1) = (\overset{x_1}{-2}, \overset{y_1}{4})$

$y - y_1 = m(x - x_1)$

$y - 4 = (-4)(x - (-2))$ 2

$y - 4 = -4x + (-4)(2)$

+4 +4

$y = -4x - 8 + 4$

$\boxed{y = -4x - 4}$

If $(x_1, y_1) > (0, -4)$

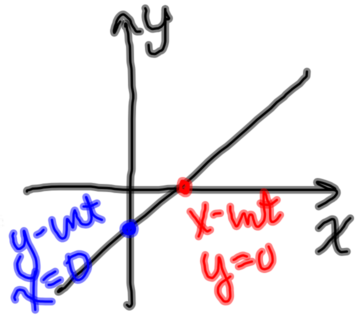
$y - y_1 = m(x - x_1)$

$y - (-4) = (-4)(x - 0)$

$y + 4 = -4x \Rightarrow \boxed{y = -4x - 4}$

When we simplify our point-slope form we are writing the line in the slope-intercept form,

$$\boxed{y = mx + b}$$



Again, m is the slope and now b is the y-intercept.

The y-intercept is the place where the line crosses the y-axis.
The x-intercept is the place where the line crosses the x-axis.

Example
Graph the line $y = -4x - 4$ and find the intercepts.

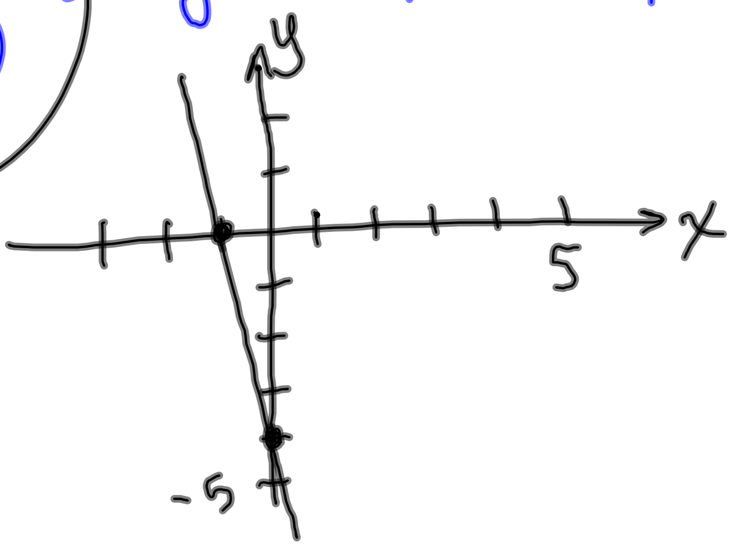
Answer

x-int is at $y=0$: $\overset{+4}{0} = -4x \overset{+4}{-4}$

$(-1, 0)$ $\quad 4 = -4x \Rightarrow x = \frac{4}{-4} = -1$

y-int is at $x=0$: $y = -4(0) - 4 = -4$

$(0, -4)$



$Ax + By = C$ is the GENERAL FORM of a line.

Example
Graph the line $3x - 4y = 12$ on paper and on the calculator.

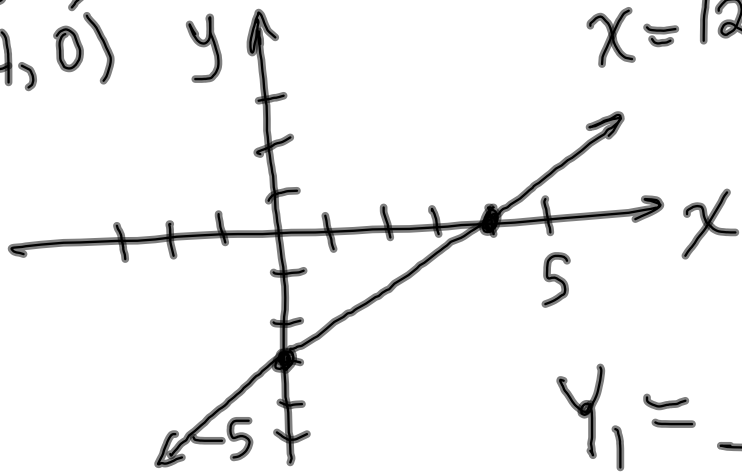
Answer

$x = 0 \Rightarrow 3(0) - 4y = 12 \Rightarrow -4y = 12 \Rightarrow$

$(0, -3)$ $y = 12/-4 = -3$

$y = 0 \Rightarrow 3x - 4(0) = 12 \Rightarrow 3x = 12$

$(4, 0)$ $x = 12/3 = 4$



$Y_1 =$ ______

$3x - 4y = 12$

$-4y = 12 - 3x$

$y = (12 - 3x)/(-4)$

Two lines are parallel if they have the same slope and different y-intercepts, $m_1 = m_2$ and $b_1 \neq b_2$

Two line are perpendicular if the product of their slopes is -1, $m_1 \cdot m_2 = -1$ or $m_1 = \frac{-1}{m_2}$

Example

Given the line L_1 is $y = 2x + 4$,

(a) find a line parallel to L_1 that passes through the point (4, 4)

(b) find a line perpendicular to L_1 that passes through the point (4,4)

Answer

$m = 2$

a) $m_1 = 2 = m_2$

$y - y_1 = m(x - x_1)$

$y - 4 = 2(x-4) = 2x - 8$

$+4 \qquad +4$

$y = 2x - 4$

b) $m_1 \cdot m_2 = -1$ or $m_2 = -\frac{1}{m_1} = -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}(x - 4) = -\frac{1}{2}x + 2$

$+4 \qquad +4$

$y = -\frac{1}{2}x + 6$

APPLICATIONS

Example 2010

In the ~~1990's~~ for wages less than the maximum taxable wage base, Social Security contributions by employees are 6.2% of the employee's wages.

a) Find a linear model that expresses the relationship between wages and Social Security contributions for employees earning less than the maximum ($106,800 in 2010). $6.2\% = 6.2/100$

b) Graph this equation and find the social security contribution for an employee earning $35,000 in wages in a year.

Answer

Let x = wages in \$ and $x \le 106{,}800$

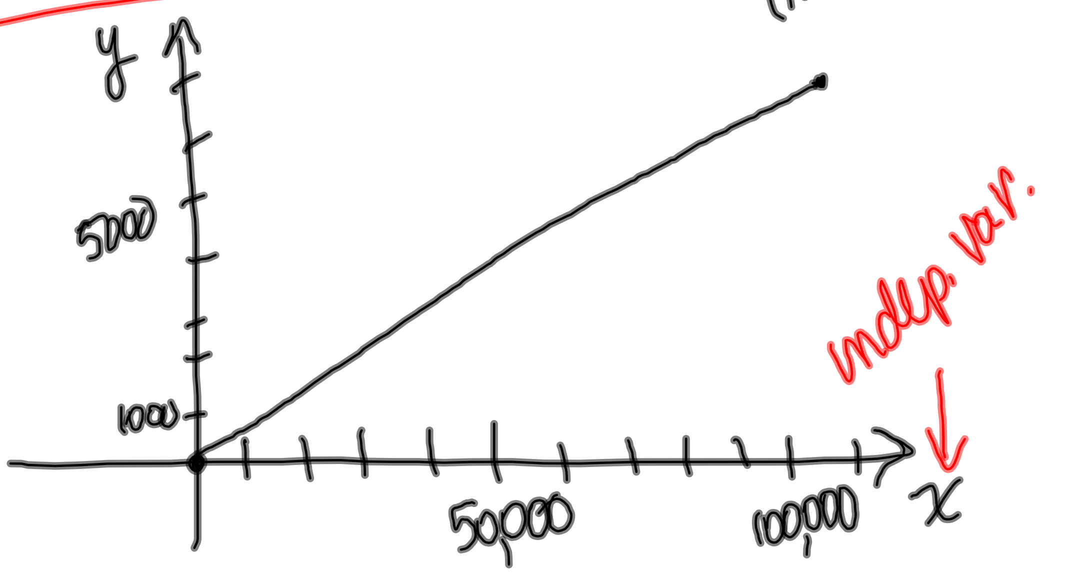
y = SS contrib. in \$

(x,y): $(0,0)$ and $(100, 100 * .062)$

$(100, 6.2)$

$$\boxed{y = 0.062x}$$

(106800, 6621.6)



$\boxed{y = mx + b}$

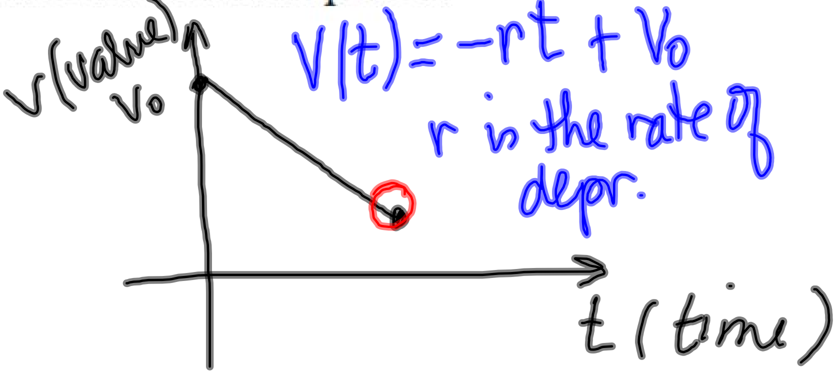
LINEAR BUSINESS MODELS

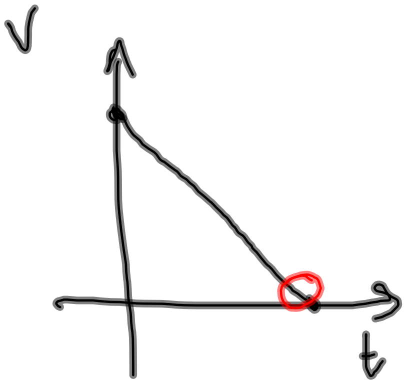
The rate of depr. of an item is given by the negative of the slope of the depr. line.

Depreciation: the value, V, of an item decreases linearly with time. The item has an initial value and then the value decreases by the same amount each time period.

$V(t) = -rt + V_0$

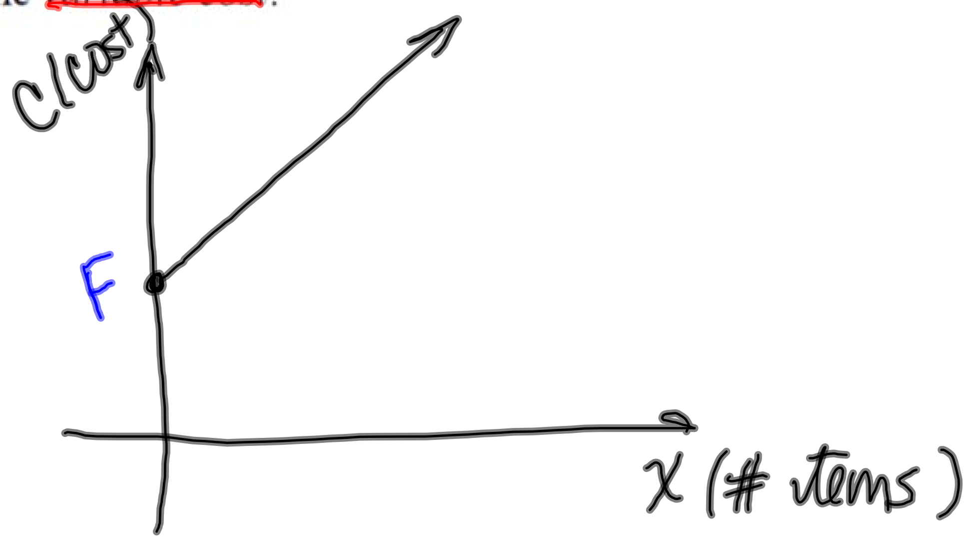
r is the rate of depr.



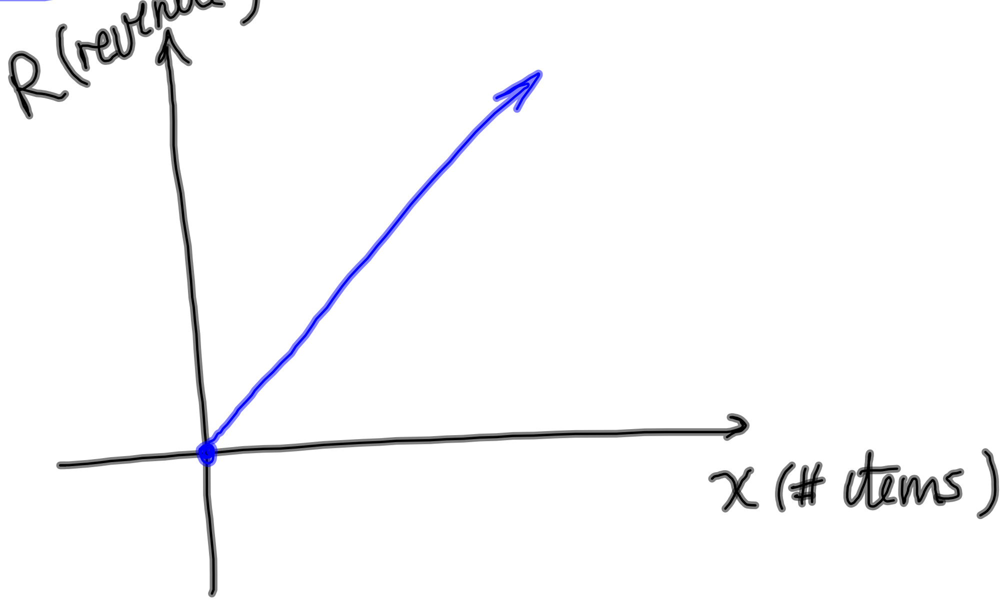


Scrap value: what the item is worth when you scrap it.

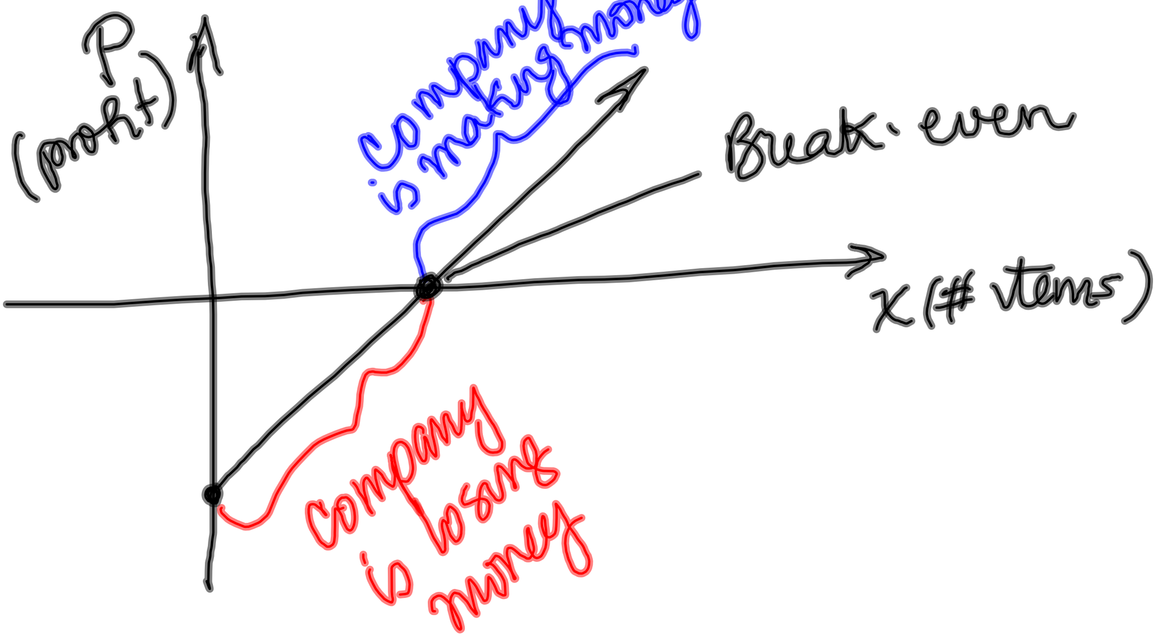
Cost: in a linear cost model the TOTAL cost to make x items is $C(x) = cx + F$. $\boxed{F \text{ represents the } \textit{fixed costs.}}$ These are the costs you have even if you make no items. c is the cost to make each unit, called the *variable cost*.



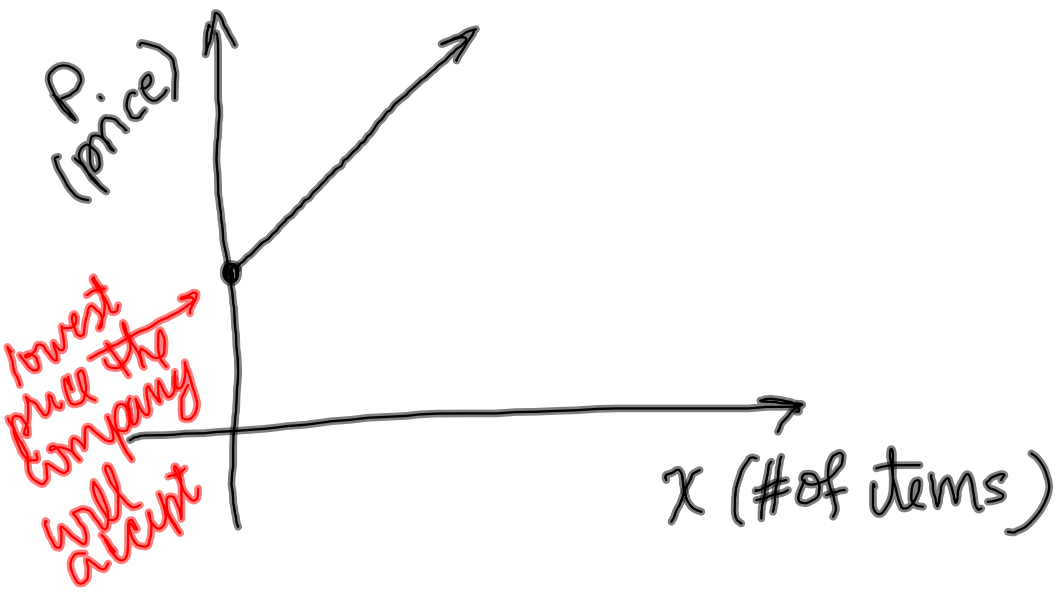
Revenue: in a linear revenue model the revenue from selling x items is $\boxed{R(x) = sx}$. s is the sale price of a single item.



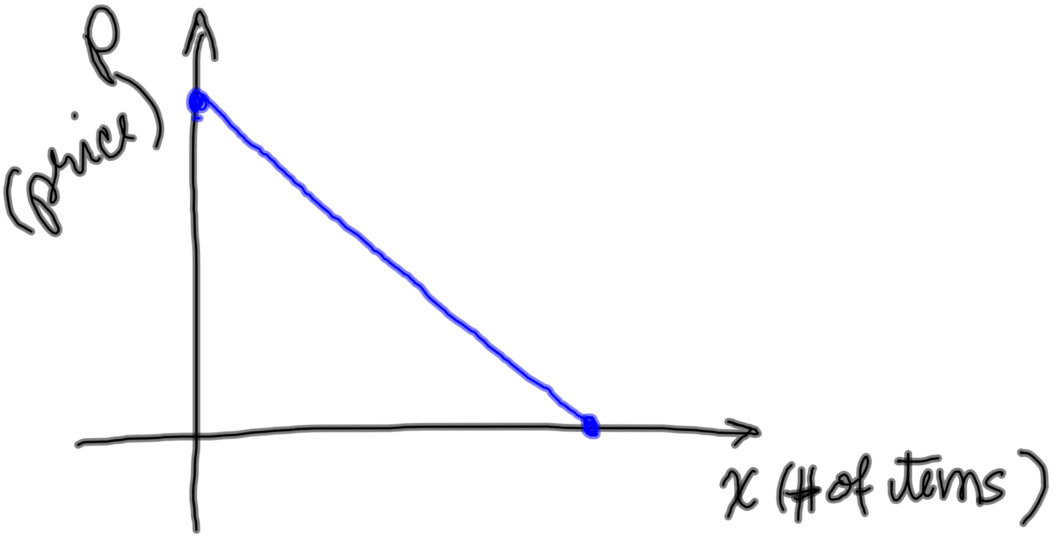
Profit: the difference between the money in (revenue) and the money spent (costs) is the profit. $P(x) = R(x) - C(x)$



Supply: in a linear supply model the number of items, x, that a company will supply at a price p is given by $S(x) = p = m_S x + b_S$.



Demand: in a linear demand model the number of items, x, that consumers will purchase at a price p is given by
$D(x) = p = m_D x + b_D$.



DEPRECIATION

alt: (0,18) and (7,4) with V in thousands of dollars

Example

A car is purchased for \$18,000 and is kept for 7 years. At the end of 7 years the car is sold for \$4000. Find an equation that models the decrease in the value of the car over time. What is the car worth after 3 years?

Answer

(t, V): (0, 18000) and (7, 4000)

where t is the time in years and V is the value in dollars

$$m = \frac{\Delta y}{\Delta x} = \frac{18000 - 4000}{0 - 7} = \frac{14000}{-7} = -2000$$

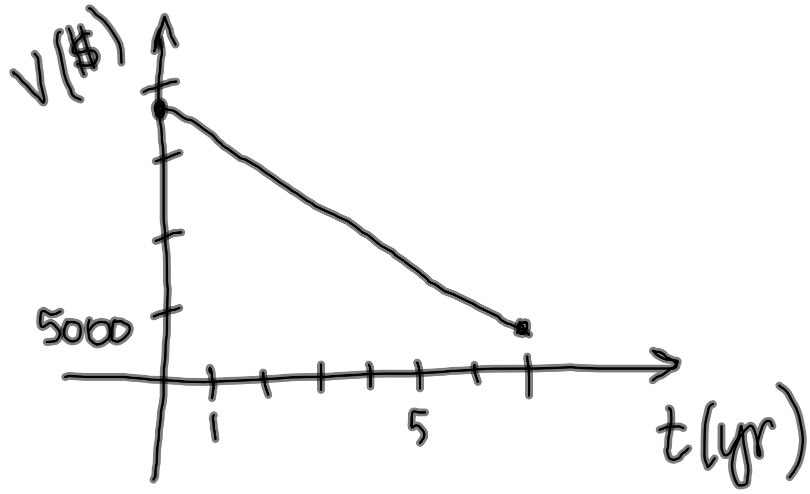
The rate of depr is 2000 \$/yr

$$y - y_1 = m(x - x_1)$$

$$y - 18000 = -2000(x - 0) = -2000x$$

$$y = -2000x + 18000$$

$$\Rightarrow \boxed{V(t) = -2000t + 18000}$$



COST, REVENUE and PROFIT

fixed and variable costs

Example

Suppose a company manufactures baseball caps. In a day they can produce 100 caps for a total cost of $600. If no caps are produced their costs are $200 per day. The caps sell for $8 each. Find the cost, revenue and profit equations. $m = \frac{600-200}{100-0} = \frac{400}{100} = 4$

Answer

cost: $(x, C) = (100, 600)$ and $(0, 200)$

$C(x) = 4x + 200$

x = # of b-ball caps produced

C = the total cost in $

revenue: $R = 8x$

x = # of b-ball caps

R = revenue in $

Profit $P = R - C$

$= 8x - (4x + 200)$

$P = 4x - 200$

x = # of b-ball caps

P = profit in $

SUPPLY AND DEMAND

Example

A baker is willing to supply 16 jumbo cinnamon rolls to a café at a price of \$1.70 each. If she is offered \$1.50 for each roll, she will supply 4 fewer rolls to the café. At the café, customers will purchase no cinnamon rolls if the cost is \$7.20 each. However, if the price of a cinnamon roll is \$0.80, the café can sell 40 of these rolls.

Find the supply and demand equations for jumbo cinnamon rolls.

Supply

$(x,p) = (16, 1.7)$ and $(16-4, 1.5)$

$(12, 1.5)$

$m = \frac{1.7-1.5}{16-12} = \frac{.2}{4} = 0.05$

$y - y_1 = m(x - x_1)$

$y - 1.5 = (.05)(x-12)$

$y = 0.05x + .9$

$$\boxed{\begin{array}{l} S(x) = p = 0.05x + .9 \\ x = \text{\# of cin. rolls supplied} \\ p = \text{price for each cin roll in \$} \end{array}}$$

Demand

$(x,p) = (0, 7.2)$ and $(40, .8)$

$m = \frac{7.2-.8}{0-40} = \frac{6.4}{-40} = -.16$

$y - y_1 = m(x - x_1)$

$y - 7.2 = -.16(x-0)$

$y = -.16x + 7.2$

$D(x) = p = -.16x + 7.20$

x = # of cin. rolls demanded

p = price per cin. roll in \$

THE INTERSECTION OF TWO LINES

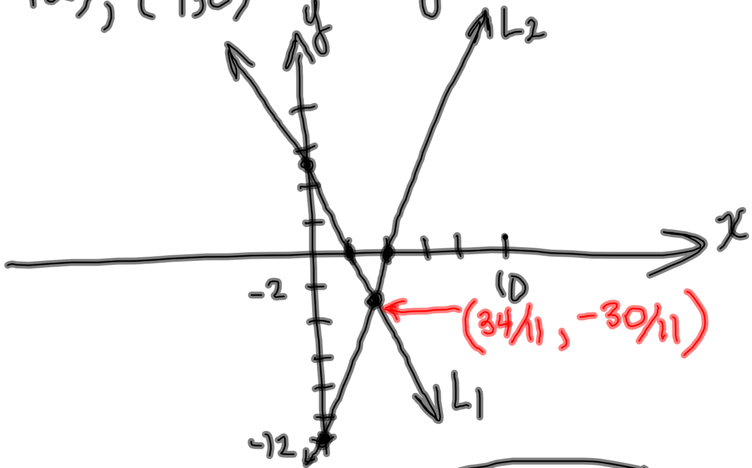
Find where the lines $10x + 4y = 20$ and $3x - y = 12$ intersect.

$L_1: 10x + 4y = 20$ $\quad x = 0 \Rightarrow 4y = 20 \Rightarrow y = 5$

$(0,5), (2,0)$ $\quad y = 0 \Rightarrow 10x = 20 \Rightarrow x = 2$

$L_2: 3x - y = 12$ $\quad x = 0 \Rightarrow -y = 12 \Rightarrow y = -12$

$(0,-12), (4,0)$ $\quad y = 0 \Rightarrow 3x = 12 \Rightarrow x = 4$



$L_1: 3x - y = 12 \Rightarrow \boxed{y = 3x - 12}$

$+y \quad +y$

$-12 \quad -12$

$L_2: 10x + 4y = 20 \; : \; 10x + 4(3x - 12) = 20$

$10x + 12x - 48 = 20$

$22x = 68 \Rightarrow x = 68/22 = 34/11$

$y = 3x - 12 = 3(34/11) - 12 = -30/11$

$(34/11, -30/11)$

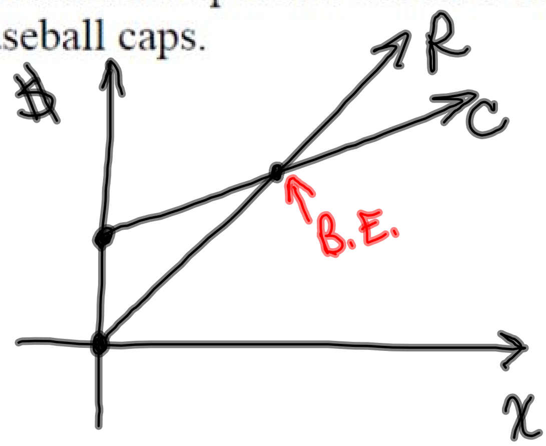
$\frac{4y}{4} = \frac{20 - 10x}{4}$

$y = 5 - 2.5x$

Break-even Point: This is where the cost to produce x items is the same as the revenue brought in from selling these x items. This occurs when $R(x)=C(x)$.

Example

Find and interpret the break-even point for making and selling baseball caps.



$R(x) = C(x)$

$8x = 4x + 200$

$-4x \quad -4x$

$4x = 200 \Rightarrow x = \frac{200}{4} = 50$

$R(50) = 8(50) = 400$

$C(50) = 4(50) + 200 = 400$

B.E at $(50, 400)$ which means that when 50 caps are made and sold, the profit is zero, and the revenue and cost for these caps is \$400.

Equilibrium Point: This is the price p that the consumer and producer are willing to pay/accept for x items. This occurs when $S(x) = D(x)$

Example
Find and interpret the $\boxed{\text{equilibrium point}}$ for the supply and demand for jumbo cinnamon rolls.

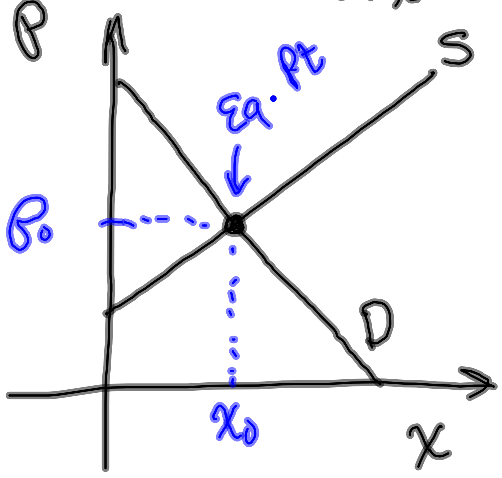
$S(x) = p = 0.05x + .9$
$D(x) = p = -0.16x + 7.2$

$S(x) = D(x)$

$0.05x + .9 = -0.16x + 7.2$
$+.16x \qquad +.16x$

$.21x + .9 = 7.2$
$-.9 \qquad -.9$

$.21x = 6.3 \Rightarrow x = \dfrac{6.3}{.21} = 30$



$S(30) = 0.05(30) + .9$
$= 2.4$

$D(30) = -.16(30) + 7.2$
$= 2.4$

Eq. pt is (30, 2.4) which means that 30 cin rolls are supplied and demanded at a price of \$2.40 each.