Applied Finite Mathematics

Tomastik/Epstein

Applied Finite Mathematics, Second Edition

Edmond C. Tomastik, University of Connecticut
Janice L. Epstein, Texas A&M University

Editor: Carolyn Crockett
Editorial Assistant: Rebecca Dashiell
Marketing Manager: Myriah Fitzgibbon
Technical Editor: Mary Kanable
Illustrator: Jennifer Tribble
Photographs: Janice Epstein

Library of Congress Control Number: 2008927157
ISBN-13: 978-0-495-55533-9
ISBN-10: 0-495-55533-9

Brooks/Cole
10 Davis Drive
Belmont, CA 94002-3098
USA

Cengage Learning is a leading provider of customized learning solutions with office locations around the globe, including Singapore, the United Kingdom, Australia, Mexico, Brazil, and Japan. Locate your local office at: **international.cengage.com/region**.

Cengage Learning products are represented in Canada by Nelson Education, Ltd.

For your course and learning solutions, visit **academic.cengage.com**. Purchase any of our products at your local college bookstore or at our preferred online store **www.ichapters.com**.

Printed in the United States of America
1 2 3 4 5 6 7 12 11 10 09 08

Preface

Applied Finite Mathematics is designed for a finite mathematics course aimed at students majoring in business, management, economics, or the life or social sciences. The text can be understood by the average student with one year of high school algebra. A wide range of topics is included, giving the instructor considerable flexibility in designing a course. Optional technology material is available where relevant.

Applications truly play a central and prominent role in the text. This is because the text is written for *users* of mathematics. Thus, for example, a concrete applied problem is presented first as a motivation before developing a needed mathematical topic. After the mathematical topic has been developed, further applications are given so that the student understands the practical need for the mathematics. This is done so consistently and thoroughly that after going completing some chapters, the student should come to believe that mathematics is everywhere. Indeed, countless applications are drawn from actual referenced examples extracted from journals and other professional texts and papers.

No other skill is more important than the ability to translate a real-life problem into an appropriate mathematical format for finding the solution. Students often refer to this process as "word problems." Whereas linear systems of equations, linear programming problems, and financial problems, for example, can easily be solved using modern technology, no calculator or computer, now or in the foreseeable future, can translate these applied problems into the necessary mathematical language. Thus students, in their jobs, will most likely use their mathematical knowledge to translate applied problems into necessary mathematical models for solution by computers.

To develop these needed skills many word problems, requiring the writing of one linear equation, are given in the introductory sections. This prepares the student for the many word problems that require creating systems of linear equations. The word problems continue in subsequent chapters, for example, on linear programming.

✧ Important Features

The text can be understood by the average student with a minimum of outside assistance. Material on a variety of topics is presented in an interesting, informal, and student-friendly manner without compromising the mathematical content and accuracy. Concepts are developed gradually, always introduced intuitively, and culminate in a definition or result. Where possible, general concepts are presented only after particular cases have been presented.

Historical Boxes Scattered throughout the text, and set-off in boxes, are historical and anecdotal comments. The historical comments are not only interesting in themselves, but also indicate that mathematics is a continually developing subject.

Connections The Connection boxes relate the material to contemporary problems. This makes the material more relevant and interesting.

Applications The text includes many meaningful applications drawn from a variety of fields. For example, every section opens by posing an interesting and relevant applied problem using familiar vocabulary, which is then solved later in the section after the appropriate mathematics has been developed. Applications are given for all the mathematics that are presented and are used to motivate the student.

Worked Examples About 300 worked examples, including about 100 self-help exercises mentioned below, have been carefully selected to take the reader progressively from the simplest idea to the most complex. All the steps needed for the complete solutions are included.

Self-Help Exercises Immediately preceding each exercise set is a set of Self-Help Exercises. These approximately 100 exercises have been very carefully selected to bridge the gap between the exposition in the chapter and the regular exercise set. By doing these exercises and checking the complete solutions provided, students will be able to test or check their comprehension of the material. This, in turn, will better prepare them to do the exercises in the regular exercise set.

Exercises The text contains over 2000 exercises. Each set begins with drilling problems to build skills, and then gradually increases in difficulty. The exercise sets also include an extensive array of realistic applications from diverse disciplines. Technology exercises are included.

End of Chapter Projects Most chapters contain an in-depth exportation of an important concept taught in the chapter. This provides strong connections to real applications or a treatment of the material at a greater depth than in the main part of the chapter.

Flexibility and Technology The text does not require any technology. However, important material on how to use technology is included. This material is tucked out of the way of a reader not interested in using technology, being placed at the end of a section as technology notes and also within green boxes in the margin.

✧ Technology

For those finite math classes that are taught with a graphing calculator or a spreadsheet, this text has abundant resources for the student and the instructor. The most accessible resource is the green margin boxes with the **Technology Option**. These are designed for students who are familiar with a graphing calculator and wish to see how the current example is worked using the calculator. For those students who need step-by-step directions, the **Technology Corner** provides details on using a graphing calculator or a spreadsheet to carry out the mathematical operations discussed in the section. While the text focuses on the use of a TI-83/84 and Microsoft Excel, other technology help is available upon request.

Student Aids

- **Boldface cyan text** is used when new terms are defined.
- **Boxes** are used to highlight definitions, theorems, results, and procedures.
- **Remarks** are used to draw attention to important points that might otherwise be overlooked.
- **Titles** for worked examples help to identify the subject.
- **Chapter summary outlines** at the end of each chapter conveniently summarize all the definitions, theorems, and procedures in one place.
- **Review exercises** are found at the end of each chapter.
- **Answers** to odd-numbered exercises and to all the review exercises are provided at the end of each chapter.
- A student's **solution manual** that contains completely worked solutions to all odd-numbered exercises and to all chapter review exercises is available.

Instructor Aids

- An **instructor's manual** with completely worked solutions to all the exercises is available free to adopters.
- **WebAssign** A selection of questions from every section of the text will be available for online homework on the WebAssign system. These homework questions are algorithmically generated and computer graded.

Content Overview

Chapter 1. An introduction to the theory of the firm with some necessary economics background is provided to take into account the students' diverse backgrounds. The next three sections cover linear systems of equations. The last (optional) section on least squares provides other examples and applications of the use of linear equations.
Chapter 2. The first three sections cover the basic material on matrices. Although many applications are included in the first three sections, the fourth (optional) section is entirely devoted to input–output analysis, which is an application of linear systems and matrices used in economics.
Chapter 3. The first section is an introduction to linear programming with the emphasis on translating applied problems into a mathematical format for solving them. The next two sections then develop the necessary mathematics needed to solve the linear programming problems. This includes

graphing systems of linear equations and using a geometric method of finding solutions. The final (optional) section considers a post-optimal analysis, including determining excess resources and shadow prices.

Chapter 4. The first two sections give an introduction to sets and counting the number of elements in a set. The third section then sets the background for probability by considering sample spaces and events. The next two sections then introduce the basics of probability and their rules. The next two sections cover conditional probability and Bayes' theorem.

Chapter 5. This chapter involves counting and probability. The first four sections cover the multiplication principle, permutations, combinations, probability applications of counting principles, and Bernoulli trials. The last (optional) section considers the binomial theorem.

Chapter 6. The first section revisits probability distributions and introduces histograms. The next two sections look at the measure of central tendency and the measure of the spread of data. The next sections consider the normal distribution, the approximation of the binomial distribution by a normal distribution, and finally the Poisson distribution.

Chapter F. This chapter covers finance. The first two sections cover simple and compound interest. The next two sections cover annuities, sinking funds, present value, and amortization. This chapter on finance does not depend on any of the other material and can be covered at any point in the course.

Chapter S. The first section presents the simplex method for standard maximization problems. The second section shows how to solve minimization problems by solving the dual problem, while the third section considers more general linear programming problems. Finally, the last section looks at post-optimal analysis.

Chapter M. The basic material on Markov processes, covering both regular and absorbing Markov processes, is presented in this chapter.

Chapter G. Game theory and its important connection to linear programming is presented in this chapter. This material gives the basics on the extensive interrelationship between linear programming and the celebrated theory of games developed by von Neumann and important in economic theory.

Chapter L. This chapter covers the basic topics in logic with an application in the last section to switching networks.

✧ Some Additional Comments on the Contents

In Chapter 1 when solving systems of linear equations with an infinite number of solutions we will have free variables as parameters. We make it clear that in a list of variables, such as x, y, z, and u, the last variable need not be the free one. Rather, any of the variables can be a free variable. This requires us to develop a solution plan that can address this issue.

Also when solving a system of linear equations with an infinite number of solutions in an applied application, the parameter may require some constraints. For example, the parameter may need to be an integer, or an even integer, or have a bound above and below. Furthermore, it is possible in an applied problem that there is no acceptable solution, even though there are an infinite number of solutions of the abstract **mathematical** system.

Suppose there are three equations and three unknowns, say x, y, and z, in a system of linear equations. When using the augmented matrix to solve the system, the normal procedure is to first reduce this matrix to a matrix with ones down the diagonal and zeros below the diagonal. Students invariably notice that we now have found the z value, so why not substitute this into the previous equation, solve for y, and then use these two values to substitute into the first equation in order to find x. This is formally called backward substitution. Since this is such a natural way of solving the system, we follow backward substitution in this text. In fact, software used to solve systems follow just this plan. (See "Matrix Computations" by Gene H. Golub and Charles F. van Loan.) It does not require any more calculations than some other methods that are sometimes taught.

We also indicate in an optional subsection that following the solving plan for systems of linear equations given in this text is actually **more** efficient and in general requires **fewer** calculations than any other solving plan found in some other texts.

Acknowledgments

At the University of Connecticut we are thankful for the support offered by Michael Neumann, Jeff Tollefson, Gerald Leibowitz, and David Gross. At Texas A&M University we are thankful for the support of G. Donald Allen and the feedback of Kathryn Bollinger, Kendra Kilmer, and Heather Ramsey were invaluable. We wish to express our sincere appreciation to each of the following reviewers for their many helpful suggestions.

Marti Mclard, University of Tennessee, Knoxville

John Herron, University of Montevallo

Fritz Keineut, Iowa State University

On a personal level, we both are grateful to our families for their patience and support.

Edmond C. Tomastik, University of Connecticut
Janice L. Epstein, Texas A&M University
April, 2008

Applied Finite Mathematics

Contents

Applied Finite Mathematics