6.1 – Sets and Set Operations

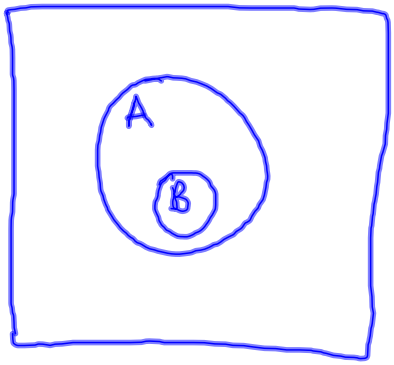
Venn Diagram notation: the problem lives here

- A rectangle represents the universal set
- Circles are sets in the universal set.

Example

$U = \{x \mid x \text{ is a pos integer less than } 17\}$ set builder notation

$= \{1, 2, 3, \ldots 16\}$ roster notation



$A = \{x \mid x \text{ is a even pos. int. less than } 17\}$

$= \{2, 4, 6, 8, 10, 12, 14, 16\}$

$B = \{x \mid x \text{ is a pos. multiple of 4 less than } 17\}$

$= \{4, 8, 12, 16\}$

$B \subseteq A$ $A \subseteq A$

$B \subset A$ $A \not\subset A$

↑ proper set ↑ a subset

$\{8\} \subset B$ $8 \in B$ ↑ is an element

WebAssign PDF ?

Given $A = \{a\}$ list all subsets : $\emptyset$, $\{a\}$ ②

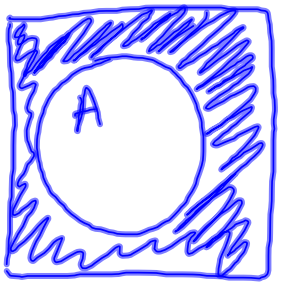
$B = \{a, b\}$ list all subsets : $\emptyset, \{a\}, \{b\}, \{a, b\}$ ④

$C = \{a, b, c\}$ list all subsets : $\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}$ $\{a, c\}, \{c, b\}, \{a, b, c\}$ ⑧

A set with n elements has 2^n subsets

Quiz Friday on 6.1 and 6.2

Given a set A and a universal set U, the elements that are in U and are **not** in A is called the **complement** of A or A^c.



$A^c = \{x \mid x \in U \text{ and } x \notin A\}$

Shade A^c

Example

From the last example, A is the set of even integers, what is A^C?

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$

$A = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$A^c = \{1, 3, 5, \ldots 15\}$

Example: Let

$U = \{x | x \text{ is a card in a standard deck of 52 playing cards}\}$

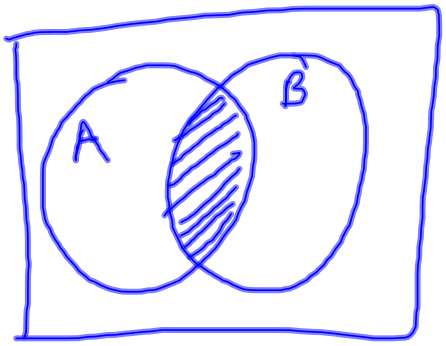
$R = \{x | x \text{ is a red card}\}$

What is R^C?

$R^C = \{x | x \text{ is a black card}\}$

$= \{x | x \text{ is not red}\}$

Those elements that belong to both A **and** B are in the **intersection** of A and B, $A \cap B$.



$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Example

Let $U = \{x|x \text{ is a card in a standard deck of 52 playing cards}\}$
$R = \{x|x \text{ is a red card}\}$
$Q = \{x|x \text{ is a queen}\}$

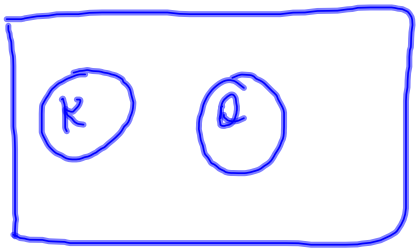
Then $R \cap Q = \{Q\heartsuit, Q\diamondsuit\} = \{Q\heartsuit, Q\diamondsuit\} = \{x \mid x \text{ is a red queen}\}$

$n(R \cap Q) = 2$

empty set

If two sets have no elements in common, that is $A \cap B = \emptyset$, then the sets are disjoint.

Let $K = \{x|x \text{ is a king}\}$ then $K \cap Q = ?$ $\emptyset = \{\}$

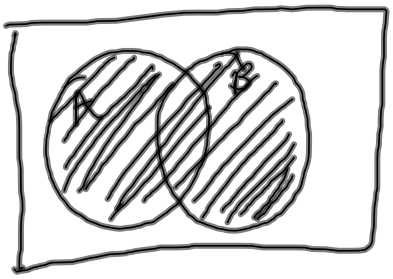


$n(K \cap Q) = 0$

$\{\emptyset\}$ is a set with one element, the empty set symbol

Those elements that belong to A **or** B are in the **union**, $A \cup B$.

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$



Note: this is the INCLUSIVE or, not the exclusive or

Example

Let $U = \{x \mid x \text{ is a card in a standard deck of 52 playing cards}\}$

$H = \{x \mid x \text{ is a heart card}\}$

$Q = \{x \mid x \text{ is a queen}\}$

$H \cup Q = \{AH, 2H, 3H, \ldots QH, KH, QD, QS, QC\}$

$n(H \cup Q) = 16$

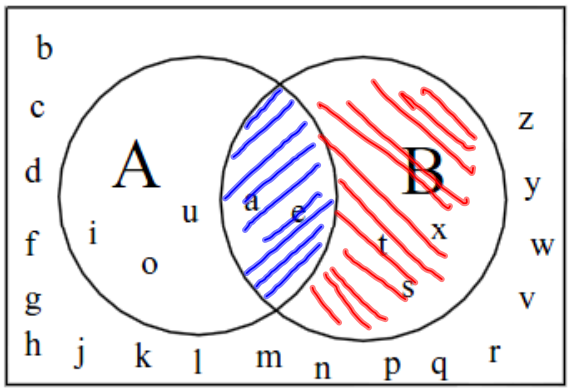
Example

Let $U = \{x | x \text{ is a letter in the English alphabet}\} = \{a, b, c, \ldots, z\}$

$A = \{x | x \text{ is a vowel}\} = \{a, e, i, o, u\}$

$B = \{x | x \text{ is a letter in the word texas}\} = \{t, e, x, a, s\}$

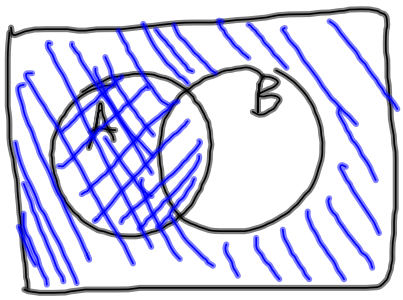
Find the following sets and express your answer in a Venn diagram



a) What is $A \cap B$? $= \{a, e\}$, $n(A \cap B) = 2$

b) What is $A^c \cap B$? $= \{t, x, s\}$

$n(A^c \cap B) = 3$



c) What is $A \cup B^c$?

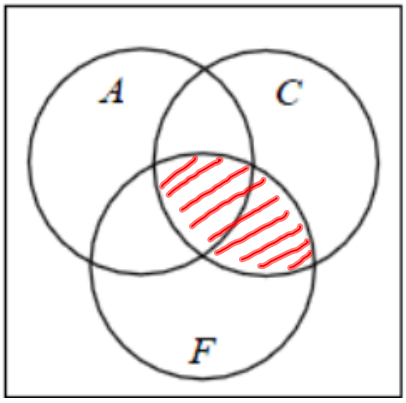
$\{a, b, c, \ldots, r, u, v, w, y, z\}$ $n(A \cup B^c) = 23$

Example

People at a home show were surveyed to see if they planned on replacing their kitchen countertops (C), their kitchen floor (F) or their kitchen appliances (A). Shade the following regions on the Venn diagram and express the region in set notation.

$C = \{x \mid x \text{ is a person replacing countertops}\}$

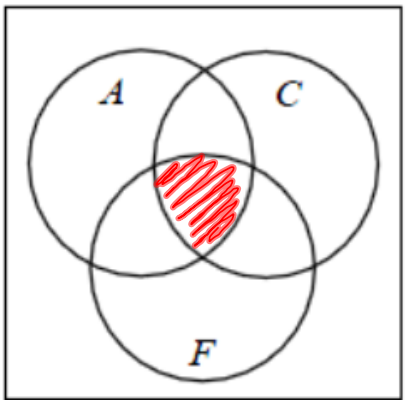
people who planned to replace their countertops and floor



$C \cap F$

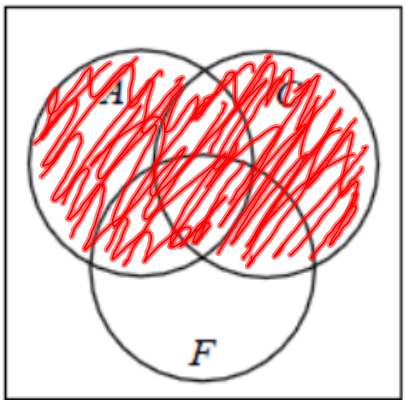
people who planned to remodel all three features

$A \cap C \cap F$



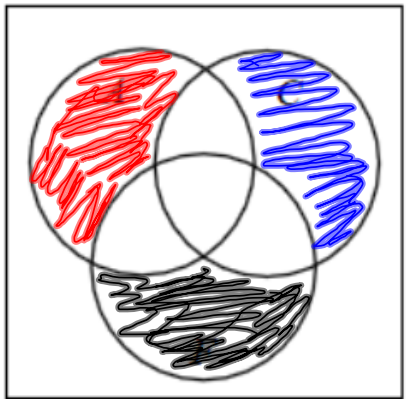
people who planned to replace their appliances or countertops

$A \cup C$



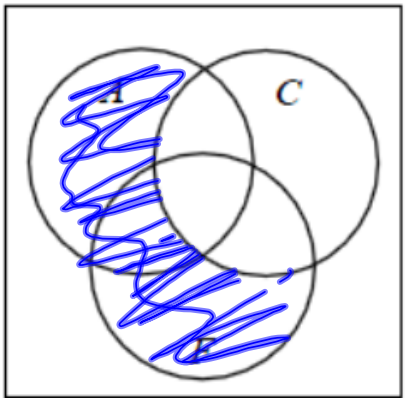
people who planned to replace exactly one of these features

$(A \cap C^c \cap F^c)$
$\cup (A^c \cap C \cap F^c)$
$\cup (A^c \cap C^c \cap F)$



people who planned to replace their kitchen appliances or floor, but not their countertops.

$(A \cup F) \cap C^c$

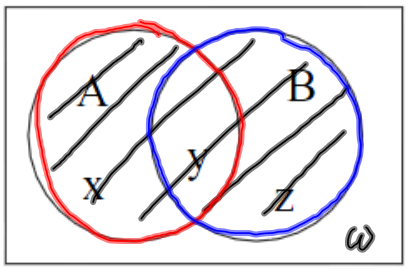


6.2 - The Number of Elements in a Finite Set

The number of elements in set A is $n(A)$.

if $A = \{x|x \text{ is a letter in the English alphabet}\}$, then $n(A)=26$.

If $A = \varnothing$ then $n(A) = 0$.



$n(A \cup B) = x+y+z$

$= x+y + y+z - y$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ union rule

Example

A store has 150 clocks in stock. 100 of these clocks have AM or FM radios. 70 clocks had FM circuitry and 90 had AM circuitry. How many had both AM and FM? How many were AM only? How many were FM only?

U

$n(U) = 150$

$n(FM \cup AM) = 100$

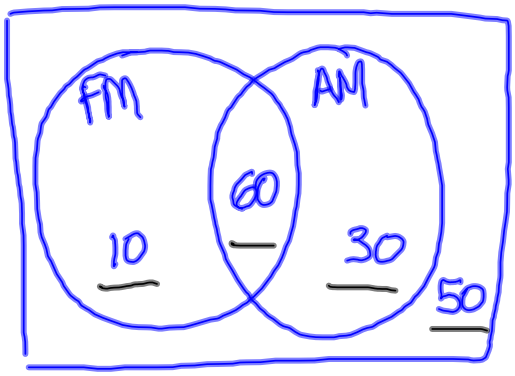
$n(FM) = 70$

$n(AM) = 90$

$100 = 70 + 90 - n(FM \cap AM)$

$n(FM \cap AM) = 60$

Answer



60 had both
10 had only FM and 80 had only AM

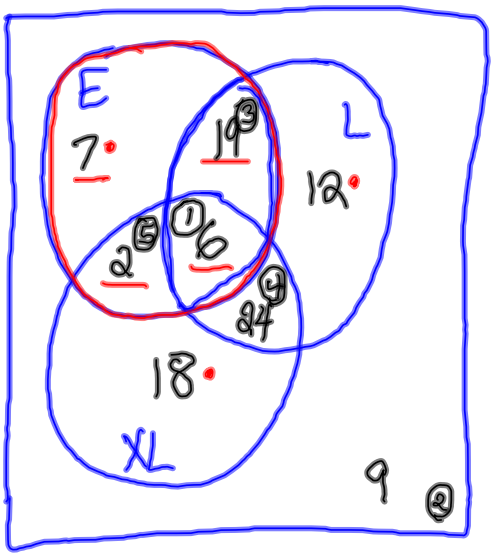
$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

UNION RULE

Example

Harvesting Fruit - We are given the following data about the contents of some delivery trucks,

34 trucks carried early peaches
61 trucks carried late peaches
50 trucks carried extra late peaches
25 trucks carried early and late peaches ③
30 trucks carried late and extra late peaches ④
8 trucks carried early and extra late peaches ⑤
1 trucks carried all three kinds of peaches ①
8 trucks carried no peaches ②



How many carried only late peaches? 12

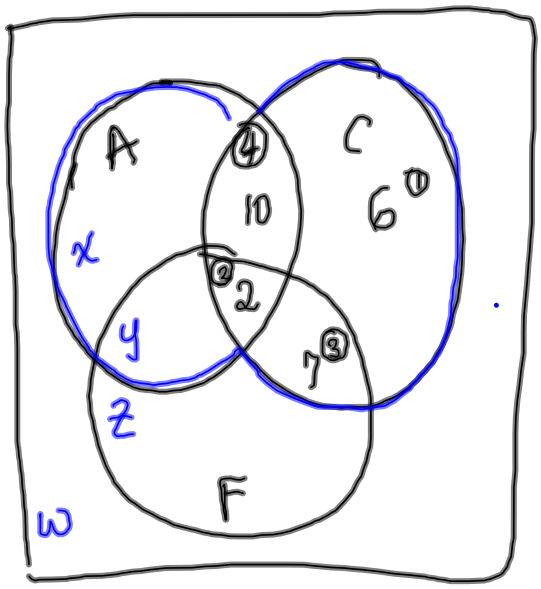
How many carried only one kind of peaches? 37

How many trucks went out? 97

Fifty-two people at a home show were surveyed to see if they planned on replacing their kitchen countertops (C), their kitchen floor (F) or their kitchen appliances (A). The following results were found:

- 22 people planned to replace exactly one of these features
- 18 people were planning to replace their kitchen appliances or floor, but not their countertops.
- 6 people planned to only replace their kitchen countertops
- 2 people planned to remodel all three features.
- 25 people planned to replace their countertops.
- 36 people planned to replace their appliances or countertops
- 9 people planned to replace their countertops and floor.

Fill in the Venn diagram with the appropriate information.



$52 = x + 10 + 6 + y + 2 + 7 + z + w$

$22 = x + 6 + z$

$18 = x + y + z$

$36 = x + 10 + 6 + y + 2 + 7$

4 eqn & 4 variables

$$\begin{aligned} x \quad\quad + z &= 16 \\ x + y + z &= 18 \\ x + y &= 11 \\ x + y + z + w &= 27 \end{aligned} \rightarrow$$

$x=9 \quad y=2 \quad z=7 \quad w=9$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 16 \\ 1 & 1 & 1 & 0 & 18 \\ 1 & 1 & 0 & 0 & 11 \\ 1 & 1 & 1 & 1 & 27 \end{array}\right] \text{ ref}$$

31 children

~~12~~ like CB

~~14~~ like PZ

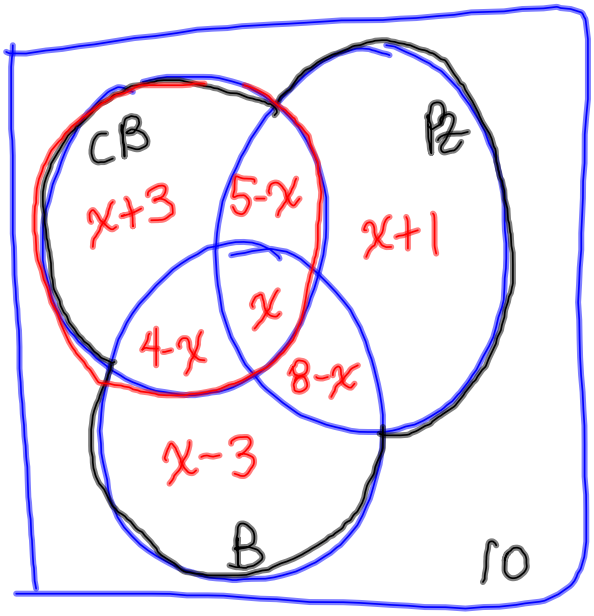
9 like B

5 like CB & PZ

~~4~~ like CB and B

~~8~~ like PZ and B

10 like none



$21 = n(CB \cup PZ \cup B)$

$21 = x+3+5-x+x+1+4-x+x+8-x+x-3$

$= x+18 \rightarrow x = 3$

One hundred shoppers are interviewed about their purchase at the store. It is found that

19 bought twinkies
37 bought diet soda
18 bought broccoli
1 bought broc, diet soda and Tw.
11 bought Twinkies and diet soda
0 bought only Twinkies and Broc
24 bought only diet soda
fill in the venn diagram