2.5 Multiplication of Matrices

Example

A flower shop sells 96 roses, 250 carnations and 130 daisies in a week. The roses sell for \$2 each, the carnations for \$1 each and the daisies for \$0.50 each. Find the revenue of the shop during the week. using matrices

Answer

$R = (96)(2) + (250)(1) + (130)(.5) = \$507$

Express the number of flowers in a 1×3 matrix:

$$A = \# \begin{bmatrix} 96 & 250 & 130 \end{bmatrix} \quad (R, C, D)$$

$$B = \begin{bmatrix} 2 \\ 1 \\ .5 \end{bmatrix} \quad (\$ \text{ ea}; R, C, D)$$

Next express the price as a 3×1 matrix:

$$A \cdot B = \# \begin{bmatrix} 96 & 250 & 130 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ .5 \end{bmatrix} = \begin{bmatrix} 96 \cdot 2 + 250 \cdot 1 + 130 \cdot .5 \end{bmatrix} = \begin{bmatrix} 507 \end{bmatrix}$$

In general, if A is $1 \times n$ and B is $n \times 1$, the product AB is a 1×1 matrix:

for app prob these labels must match.

$$A \cdot B = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \end{bmatrix} \cdot \begin{bmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{n1} \end{bmatrix} = \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + \dots + a_{1n} \cdot b_{n1} \end{bmatrix}$$

$(m \times n) \cdot (n \times p) = (m \times p)$

If A is an $m \times n$ matrix and B is a $n \times p$ matrix, then the product matrix $A \cdot B = C$ is an $m \times p$ matrix.

$$A \cdot B = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & & a_{mn} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1p} \\ b_{21} & b_{22} & \cdots & b_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{np} \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} + \ldots + a_{1n} \cdot b_{n1} & (ab)_{12} & \cdots & (ab)_{1p} \\ (ab)_{21} & (ab)_{22} & \cdots & (ab)_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ (ab)_{m1} & (ab)_{m2} & \cdots & (ab)_{mp} \end{bmatrix}$$

$(ab)_{12}$: use row 1 of A, use col 2 of B

$(ab)_{1p}$: use row 1 of A, use col p of B

$(ab)_{21}$: use row 2 of A, use col 1 of B

Matrix multiplication is not commutative. In general, $AB \neq BA$

Example
Find the products AB and BA where

$$A = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} (1)(-1)+(0)(0) & (1)(2)+(0)(-3) \\ (-2)(-1)+3(0) & (-2)(2)+(3)(-3) \end{bmatrix}$$

$(2 \times 2) \cdot (2 \times 2) = (2 \times 2)$

$$= \begin{bmatrix} -1 & 2 \\ 2 & -13 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} -5 & 6 \\ 6 & -9 \end{bmatrix} \neq AB$$

$1 \cdot A = A$

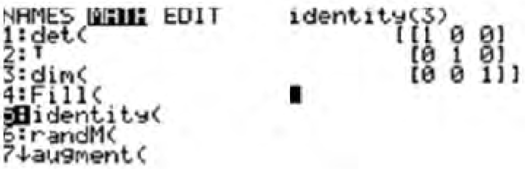
One special matrix is called the identity matrix, I.
It is a square matrix with 1's on the diagonal and zeros elsewhere,

$$I = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

$A \cdot I = A = IA$

1? I, 1, l

I_2 is a 2×2 identity matrix and I_n is an $n \times n$ identity matrix.



The identity matrix has the following property

$A \cdot I = A = I \cdot A$

Matrix multiplication and linear equations:

Example

Write the following system of linear equations as a matrix equation

$$2x - 3y = 6$$
$$-x + 2y = 4$$

Answer

(2x2) · (2x1) → 2x1

$$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix} = \begin{bmatrix} 2x + (-3)y \\ -1x + 2y \end{bmatrix} \Rightarrow$$

row1 col1: $6 = 2x - 3y$
row2 col1: $4 = -x + 2y$

$A \cdot X = B$

ALWAYS SOLVE SYS USING RREF

2.6 Inverse of a Square Matrix

$2\overline{)1.0}$

For any non-zero real number r, the reciprocal (or inverse) is $\frac{1}{r}$ or r^{-1}

Multiplicative identity: $2 \cdot \frac{1}{2} = 1 = 2 \cdot 2^{-1} = 1$

all $\mathbb{R}$ have an inverse except zero

For matrices, the inverse is A^{-1} and it is defined by

$$A^{-1} \cdot A = I = A^{-1} A$$

A matrix with no inverse is called singular.

If needed, find the inverse with the x^{-1} function on the calculator.

The one use of matrix inverses is to solve matrix equations.

Solve the matrix equation $AX = B$ for X

$$\frac{ax}{a} = \frac{b}{a} \Rightarrow x = \frac{b}{a}$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

$$\boxed{X = A^{-1}B}$$

~~AXA^{-1}~~ → mess

Solve the matrix equation $D = X - AX$ for X.