

## Using Leontief Input-Output Models

### Part I

The New Millennium Village has an economy consisting of three sectors: agriculture (*a*), manufacturing (*m*), and transportation (*t*). The input-output matrix is given by

$$A = \begin{matrix} & \begin{matrix} a & m & t \end{matrix} \\ \begin{matrix} a \\ m \\ t \end{matrix} & \begin{bmatrix} 0.3 & 0.25 & 0.05 \\ 0.2 & 0.4 & 0.45 \\ 0.15 & 0.35 & 0.5 \end{bmatrix} \end{matrix}$$

When you read an input-output matrix, the columns represent one unit of a resource being produced, requiring the given number of units of each resource listed in that column. In other words, the **OUTPUTS** are the headings of the *columns* and the **INPUTS** required to produce the outputs are the labeled *rows*.

1. The production of 1 unit of agriculture requires  
0.3 units of agriculture, \_\_\_\_\_ units of manufacturing, and 0.15 units of \_\_\_\_\_.
  
2. The production of 1 unit of manufacturing requires  
\_\_\_\_\_ units of agriculture, \_\_\_\_\_ units of manufacturing, and \_\_\_\_\_ units of transportation.
  
3. If 1 unit = \$1000 worth of a resource, then the production of \$1000 worth of agriculture requires
  - a.  $\frac{0.3}{\text{units of } a \text{ needed to make a unit of } a} \times \frac{1000}{\text{value of a unit of } a} = \$300 \text{ worth of agriculture}$
  - b. \_\_\_\_\_ · 1000 = \$\_\_\_\_\_ worth of manufacturing
  - c. \_\_\_\_\_ · \_\_\_\_\_ = \$\_\_\_\_\_ worth of transportation
  
4. If 1 unit = \$1,000,000 worth of a resource, then the production of \$3,000,000 worth of transportation, which equals \_\_\_\_\_ units of transportation, requires
  - a.  $\frac{0.05}{\text{units of } a \text{ needed to make a unit of } t} \times \frac{3}{\text{units of } t} \times \frac{1,000,000}{\text{value of a unit of } a} = \$ \text{_____} \text{ worth of agriculture}$
  - b. \$ \_\_\_\_\_ worth of manufacturing
  - c. \$ \_\_\_\_\_ worth of transportation

**Part II**

**An economy consists of three sectors: crafting ( $C$ ), fishing ( $F$ ) and gathering ( $G$ ). The input-output matrix for this economy is given by**

$$A = \begin{matrix} & \begin{matrix} C & F & G \end{matrix} \\ \begin{matrix} C \\ F \\ G \end{matrix} & \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.3 \\ 0.2 & 0.2 & 0.1 \end{bmatrix} \end{matrix}$$

1. What is the value of the entry  $a_{12}$ ? \_\_\_\_\_
2. The value of  $a_{12}$  means that the production of 1 unit of \_\_\_\_\_ requires \_\_\_\_\_ units of \_\_\_\_\_.
3. How much of each resource should be produced by this economy in order to meet an external demand of 400 units of crafted products, 1200 units of fishing, and 200 units of gathered goods?

Begin by defining the following variables,

$c$  = number of units of crafting produced

$f$  = \_\_\_\_\_

$g$  = \_\_\_\_\_

The production matrix is  $X = \begin{bmatrix} c \\ f \\ g \end{bmatrix}$ . The external demand matrix is  $D = \begin{bmatrix} \text{_____} \\ \text{_____} \\ \text{_____} \end{bmatrix}$ .

Solve for  $X$  using the formula  $X = (I - A)^{-1}D$ .

$$X = (I - A)^{-1}D = \begin{bmatrix} \text{_____} \\ \text{_____} \\ \text{_____} \end{bmatrix}$$

The economy should produce \_\_\_\_\_ units of crafting, \_\_\_\_\_ units of fishing, and \_\_\_\_\_ units of gathering.

4. How much of each resource is consumed internally by this economy when meeting the aforementioned external demand? *Hint:* The total demand is the internal + external demand.

The economy internally consumes \_\_\_\_\_ units of crafting, \_\_\_\_\_ units of fishing, and \_\_\_\_\_ units of gathering.