

**Part I**

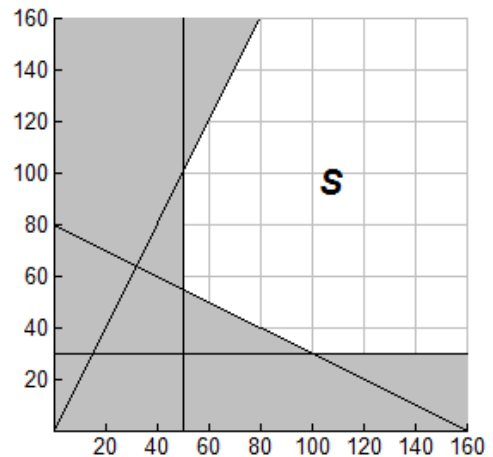
An apartment complex is being developed that will offer renters the choice of a one- or two-bedroom apartment. Based upon market research, the property owner has decided that half of the number of two-bedroom apartments is to be less than or equal to the number of one-bedroom apartments. To ensure that the complex has enough units to meet demand, at least 50 one-bedroom units and 30 two-bedroom units must be built. Each one-bedroom unit requires 4 panes of glass for windows, while each two-bedroom unit requires 8 panes. Due to pressure from the supplier, at least 640 panes of glass must be used. The cost to build a one-bedroom apartment is \$67,000 and the cost to build a two-bedroom apartment is \$81,000. Determine the number of units of each type that must be constructed in order to minimize the cost of building all units.

Let  $x$  = the number of one-bedroom apartments  
 Let  $y$  = the number of two-bedroom apartments  
 Let  $C$  = the total cost of building all units

OBJECTIVE: Minimize  $C = 67000x + 81000y$

SUBJECT TO:

- $4x + 8y \geq 640$  panes of glass
- $\frac{1}{2}y \leq x$  ratio of two to one bedroom apartments
- $x \geq 50$  min # of one-bedroom apartments
- $y \geq 30$  min # of two-bedroom apartments



1. Is the feasible region,  $S$ , bounded or unbounded? \_\_\_\_\_  
 Is it possible to find a minimum value of the objective function,  $C$ , on this feasible region? \_\_\_\_\_  
 Why or why not? *Hint*: See Theorem 2 for Linear Programming. \_\_\_\_\_

2. a. Since a minimum value of  $C$  exists, organize the corner points of the feasible region in a table and evaluate the objective function at each corner point to determine the minimum value and where it occurs.

Corner Point	$C = 67000x + 81000y$
_____	_____
_____	_____
_____	_____

- b. Complete the sentence: A minimum cost of \_\_\_\_\_ is obtained when \_\_\_\_\_ one-bedroom apartments and \_\_\_\_\_ two-bedroom apartments are constructed.
3. How many panes of glass are used at the minimum? \_\_\_\_\_
4. Is it possible to identify a maximum cost? How does this relate to Theorem 2?

**Part II**

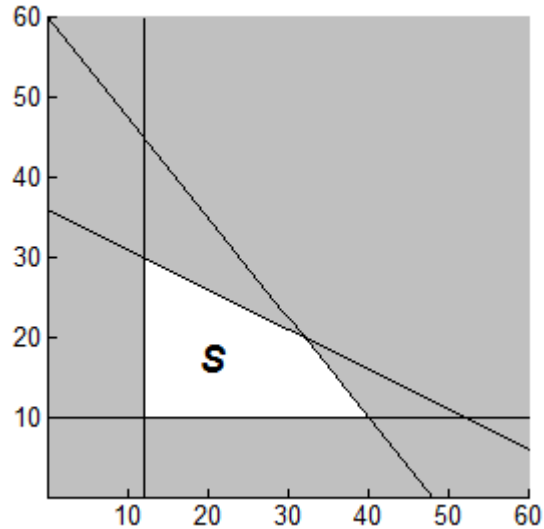
A pet store sells large angelfish for \$20 each and silver dollarfish for \$16 each. Each angelfish requires 5 hours of care and 4 ounces of flake food to reach full size, and each silver dollarfish requires 4 hours of care and 8 ounces of flake food to reach full size. The pet store has 288 ounces of food and 240 hours of care available to raise the two kinds of fish. The store wants to sell a minimum of 12 angelfish and 10 silver dollarfish. If the store wishes to maximize the revenue obtained from selling angelfish and silver dollarfish, how many of each kind of fish should be raised to full size? What is the maximum revenue?

Let  $x$  = the number of angelfish  
 Let  $y$  = the number of silver dollarfish  
 Let  $R$  = the revenue earned from the sale of fish

OBJECTIVE : Maximize  $R = 20x + 16y$

SUBJECT TO :

- $4x + 8y \leq 288$  oz. of food available
- $5x + 4y \leq 240$  hrs. of care available
- $x \geq 12$  min. # of angelfish
- $y \geq 10$  min # of dollarfish



- Is the feasible region,  $S$ , bounded or unbounded? \_\_\_\_\_  
 Is it possible to find a maximum value of  $R$  on this feasible region? Why or why not? *Hint:* See Theorem 2.

- Since a maximum value of  $R$  exists, find the revenue earned at each corner point.

Corner Point	$R = 20x + 16y$
(12, 10)	_____
(12, 30)	_____
(32, 20)	_____
(40, 10)	_____

- On the graph of the feasible region, note the two points that have the same maximum revenue. Imagine that there is a heavy line that connects these two points. What is the value of  $R$  on every point of this line segment? \$ \_\_\_\_\_

4. The maximum revenue is  $R = \underline{\hspace{2cm}}$  and is obtained at every point on the line segment joining the points  $\underline{\hspace{2cm}}$  and  $\underline{\hspace{2cm}}$ .

The equation of the line segment on which the maximum revenue lies is  $R = 20x + 16y$ .

This means that  $R = \underline{\hspace{2cm}} = 20x + 16y$ . Now write this equation for the maximum revenue line in slope-intercept form:  $\underline{\hspace{4cm}}$

The maximum revenue of \$  $\underline{\hspace{2cm}}$  occurs at all points along the line  $y = \underline{\hspace{2cm}}$  with  $\underline{\hspace{1cm}} \leq x \leq \underline{\hspace{1cm}}$ .

If there was no meaning attached to the variables in this problem, then  $x$  and  $y$  could take on *infinitely many* values on the line segment above. However, since the pet store sells only *whole numbers* of fish, only some of the points on the line segment make sense in the problem.

5. Give all the feasible options for the number of fish to be raised and sold that would give the maximum revenue. For each option, calculate how much food and care were actually used to raise that number of fish and determine any leftovers.

(Hint:  $x$  and  $y$  must each be whole numbers. Consider the possible whole-number values of  $x$  using your answer to 4. Plug these values into the slope-intercept form of the line to determine the values of  $x$  that will correspond to whole-number values of  $y$ . There are three whole-number  $(x, y)$  pairs.)

Enter the points from smallest  $x$  value to largest  $x$  value

$(x, y)$	Food Used	Leftover Food	Hours of Care Used	Leftover Hours
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>
<hr/>	<hr/>	<hr/>	<hr/>	<hr/>