Last Name: ______________ First Name: ______________ Section 141-____ UIN: _________

**Solving Systems of Linear Equations – Part 2**

**1.** A system of linear equations may have no solution.

**a.** The following augmented matrix represents a system of linear equations. Use your calculator to simplify the system.

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 20 \\ 0 & -1 & 4 & 50 \\ 1 & 3 & -1 & 10 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{array}\right]$$

**b.** Write the system of three equations corresponding to the simplified system.

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Notice that the third equation is NEVER true. This is called a contradiction.

*Every time you reach a contradiction, the system has no solution.*

**2.** A system of linear equations may have a parametric solution.
**a.** The following augmented matrix represents a system of linear equations. Use your calculator to simplify the system.

$$\begin{array}{ccc} x & y & z \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 20 \\ 0 & -1 & 4 & 50 \\ 2 & 3 & 10 & 90 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ & \_\_\_\_ & \_\_\_\_ \end{array}\right]$$

**b.** Write the system of three equations corresponding to the simplified system.

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Notice that the third equation is ALWAYS true, giving you no specific information about the values of the variables or the number of solutions to the system.

**c.** Are there any contradictions? ______

**d.** Is there a single numerical value for each variable, giving us one unique solution to the system? ____

**e.** There is no contradiction and no unique solution, so there must be *infinitely many solutions*.

It is impossible to list these infinitely many solutions. The solution to the system requires the introduction of at least one parameter (variable). The easiest way to parameterize the solution is to set a parameter for any variable of the system whose column in the reduced matrix does not have a leading one.

Note which column of the reduced matrix in **a.** (shown again below) that does *not* have a leading one.

$$\begin{array}{ccc|c} x & y & z & \\ 1 & 0 & 11 & 120 \\ 0 & 1 & -4 & -50 \\ 0 & 0 & 0 & 0 \end{array}$$

To which variable does this column correspond? ______

**f.** Next, introduce a parameter for this variable: Let $z = t$, where $t$ is any real number.

- Your goal is to write all remaining variables in terms of $t$.
- Make the substitution $z = t$ into the first two equations from **b.**:

$$x + 11\_\_\_ = 120$$
$$y - 4\_\_\_ = -50$$

Solve the first equation for $x$:

$$x + 11t = 120$$
$$x = \_\_\_\_\_\_\_\_\_\_\_\_$$

Solve the second equation for $y$:

$$y - 4t = -50$$
$$y = \_\_\_\_\_\_\_\_\_\_\_\_$$

**g.** The solution to the system (which will contain the parameter) is

($x$, $y$, $z$) = ( ___________ , ___________, $t$ ), where $t$ is __________.

**h.** A **particular solution** (or **specific solution**) to the system is obtained by choosing a particular value for the parameter, $t$.

For example, the specific solution corresponding to $t = 0$ is

$$(x, y, z) = \left(120 - 11(0), -50 + 4(0), (0)\right) = (120, -50, 0)$$

Find the specific solution corresponding to $t = 1$.

(________, ________, ________)

Find any other specific solution to the system.

Your value of $t$: ________

Your specific solution: (________, ________, ________)

Is (10, 10, 10) a specific solution to the system? _____