MATHEMATICS OF FINANCE

ON THE CALCULATOR, have TVM Solver

```
CALC VARS
1:TVM Solver…
2:tvm_Pmt
3:tvm_I%
4:tvm_PV
5:tvm_N
6:tvm_FV
7↓npv(
```

```
N=■
I%=0
PV=0
PMT=0
FV=1
P/Y=1
C/Y=1
PMT:END BEGIN
```

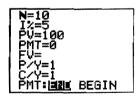N is #of compounding periods

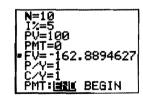I% is annual interest rate as a %

PV is Present Value

PMT is Payments

FV is Future Value

P/Y is ⎫
       ⎬ compounding frequency
C/Y is ⎭

set PMT: END (make payments at the end of the cycle)

10 yrs, annual, $100 @ 5%

To use the TVM solver, enter all known values (5 of the 6). Put the cursor on the unknown and press SOLVE (alpha – enter)

```
N=10
I%=5
PV=100
PMT=0
FV=
P/Y=1
C/Y=1
PMT:END BEGIN
```
SOLVE
```
N=10
I%=5
PV=100
PMT=0
•FV=-162.8894627
P/Y=1
C/Y=1
PMT:END BEGIN
```

NOTE about sign change – it is cash inflow and outflow.

Account has $162.89

Interest earned   162.89
                  -100.00
                  ───────
                    62.89

SIMPLE INTEREST? $50

EXAMPLE: You are planning a trip to Florida in 2 years. You want $2000 available. You find an investment paying 10% compounded quarterly. How much do you need to invest now to have the money ready in 2 years?
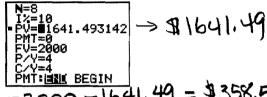
N = 2×4=8   PMT = 0   I = 10

FV = 2000   PV = ?   P/Y = 4

```
N=8
I%=10
PV=■
PMT=0
FV=2000
P/Y=4
C/Y=4
PMT:END BEGIN
```
```
N=8
I%=10
•PV=-1641.493142
PMT=0
FV=2000
P/Y=4
C/Y=4
PMT:END BEGIN
```
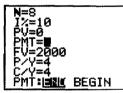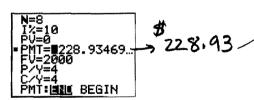→ $1641.49

Interest earned = 2000 – 1641.49 = $358.51

interest earned = 2000 – 8(228.93)
                = $168.56

Save up for the trip by making regular quarterly payments into an account paying 10% interest compounded quarterly.

N = 2×4=8   PMT = ?   I = 10

FV = 2000   PV = 0   P/Y = 4

```
N=8
I%=10
PV=0
PMT=■
FV=2000
P/Y=4
C/Y=4
PMT:END BEGIN
```
```
N=8
I%=10
PV=0
•PMT=-228.93469…
FV=2000
P/Y=4
C/Y=4
PMT:END BEGIN
```
→ $228.93

CONTINUOUS COMPOUNDING? A=Pe^{rt}

$A = 100e^{(.05 \times 10)} = 164.87$

⇒ $64.87 in interest

ANNUITIES

An annuity is an account to which regular payments are made.

An annuity that is certain and simple has the following properties:

1. The payments are made at fixed time intervals

2. The periodic payments are of equal size

3. The payments are made at the end of the interval

4. The interest is paid at the end of the interval

Many loans and savings plans are certain and simple annuities

EXAMPLE: You purchase a car for no money down and payments of $299 a month for 60 months with interest of 12% charged on the unpaid balance every month. What was the cash price of the car? How much did you pay in interest?

N = 60   PMT = 299   I = 12

FV = 0   PV = ?   P/Y = 12

```
N=60
I%=12
PV=█
PMT=299
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

```
N=60
I%=12
▪PV=█13441.55648
PMT=299
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```
→ cash price
(loan amt)
$13442

Interest paid = (60)(299) − 13442
= $4498

What happens with a 4 year (48 payments) loan?

N = 48   PMT = ?   I = 12

FV = 0   PV = 13442   P/Y = 12

```
N=48
I%=12
PV=13442
PMT=█
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

```
N=48
I%=12
PV=13442
▪PMT=█353.97941...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```
→ PMT $353.98

Interest paid = (48)(354) − 13442
= $3550

You deposit $500 per year for into a college fund paying 7% compounded annually. How much is available in 18 years? How much interest is earned?

$N = 18 \qquad PMT = 500 \qquad I = 7$

$FV = ? \qquad PV = 0 \qquad P/Y = 1$

```
N=18
I%=7
PV=0
PMT=500
FV=■
P/Y=1
C/Y=1
PMT:END BEGIN
```

```
N=18
I%=7
PV=0
PMT=500
■FV=■16999.51626
P/Y=1
C/Y=1
PMT:END BEGIN
```
$\rightarrow \$17000$

$\$17000 - 18(500) = \$8000$

You deposit $2000 per year into a retirement fund. If the money is deposited once per year in an account paying 10% compounded annually, how much is in the account after 10, 20, 30 and 40 years?
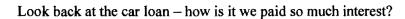
$N = 10 \qquad PMT = 2000 \qquad I = 10$

$FV = ? \qquad PV = 0 \qquad P/Y = 1$

After 10 years, $\$31,875$

After 20 years, (N=20) $\$114,550$

After 30 years, (N=30) $\$328,988$

After 40 years, (N=40) $\$885,185$

Look back at the car loan – how is it we paid so much interest?

At the end of the 1st period we owe interest on the outstanding balance of $13442.

Monthly interest rate is $12\dfrac{\%}{year} \times \dfrac{1\ year}{12\ months} = 1\dfrac{\%}{month}$

Interest owed = $13442 \times .01 = 134.42$

Principal paid = $299 - 134.42 = 164.58$

So we now owe

$13442 - 164.58 = 13277.42$

$(164.58 + (13277.42)$

EQUITY:
How much of the item that belongs to you (not the bank)

End of the 2nd period, owe the bank $13,277.42

Interest owed = $13277.42 \times .01 = 132.77$

Principal paid = $299 - 132.77 = 166.23$

Now we owe 13111.19

Equity = $13442 - 13111.19 = 330.81$

In general,

EQUITY = VALUE OF ITEM – WHAT YOU OWE THE BANK.

This can be summarized in an *AMORTIZATION TABLE*:

end of period	payments remaining	PMT	to interest	towards principal	outstanding principal	equity
0	60				13442.00	0.00
1	59	299	134.42	164.58	13277.42	164.58
2	58	299	132.77	166.23	13111.19	330.81
3	57	299	131.11	167.89	12943.31	498.69
4	56	299	129.43	169.57	12773.74	668.26
5	55	299	127.74	171.26	12602.48	839.52
6	54	299	126.02	172.98	12429.50	1012.50
55	5	299	17.34	281.66	1451.94	11990.06
56	4	299	14.52	284.48	1167.46	12274.54
57	3	299	11.67	287.33	880.14	12561.86
58	2	299	8.80	290.20	589.94	12852.06
59	1	299	5.90	293.10	296.84	13145.16
60	0	299	2.97	296.03	0.81	13441.19

(actually will be 299+0.81=299.81)

To do a line of this in the calculator:

1. Calculate the payments.

2. Change N to the *number of payments remaining on the loan*

3. Solve for PV. This is what you still how the bank (outstanding principal)

4. Equity = value of item – what you owe the bank.

EXAMPLE
You buy a $120,000 house. You make a $20,000 down payment and finance the remainder at 7.5% interest compounded monthly on the outstanding balance for 30 years.

a) How large are the monthly payments?
b) How much interest is paid in all?
c) What is the equity after 1 year? 5 years? 15 years?

$N = 30 \times 12 = 360$ PMT = ? $I = 7.5$

FV = 0 PV = 100000 P/Y = 12
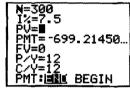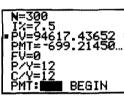
```
N=360
I%=7.5
PV=100000
PMT=█
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

```
N=360
I%=7.5
PV=100000
●PMT=█699.21450...
FV=0
P/Y=12
C/Y=12
PMT:END BEGIN
```

a) $ 699.21

b) $(360)(699.21) - 100000$
$= 151,915.60$

c) What is the equity after 1 year

```
N=348
I%=7.5
PV=■
PMT=-699.21450…
FV=0
P/Y=12
C/Y=12
PMT:■■■ BEGIN
```

```
N=348
I%=7.5
•PV=99078.16557
PMT=-699.21450…
FV=0
P/Y=12
C/Y=12
PMT:■■■ BEGIN
```

Equity = 120,000 − 99,078 = $20,922

Equity after 5 years?

```
N=300
I%=7.5
PV=■
PMT=-699.21450…
FV=0
P/Y=12
C/Y=12
PMT:■■■ BEGIN
```

```
N=300
I%=7.5
•PV=94617.43652
PMT=-699.21450…
FV=0
P/Y=12
C/Y=12
PMT:■■■ BEGIN
```

Equity = 120,000 − 94,617 = $25,383

Equity after 15 years?

```
N=180
I%=7.5
PV=■
PMT=-699.21450…
FV=0
P/Y=12
C/Y=12
PMT:■■■ BEGIN
```

```
N=180
I%=7.5
•PV=75426.66514
PMT=-699.21450…
FV=0
P/Y=12
C/Y=12
PMT:■■■ BEGIN
```

Equity = 120,000 − 75,427 = $44,573

How long to double your investment?

$10,000 at 6% annual interest compounded daily.

N = ?   I = 6   PV = 10000
PMT = 0   FV = −20000   P/Y = 365

N = 4217 days

For I = 9, find N = 2811 days

Rent-to-Own a cello: A cello is $574 to buy or $40.58 on a 24 month rent to own plan. What is the interest rate?

N = 24   I = ?   PV = −574
PMT = 40.58   FV = 0   P/Y = 12

I = 56.96 → 56.96%

Pay off Credit Card: You owe $6000 on a credit card that charges 18% annual interest compounded monthly on the outstanding principal. Make monthly payments of $120. How long to pay off? How much interest is paid in all?

N = ?   I = 18   PV = −6000
PMT = 120   FV = 0   P/Y = 12

→ 93.11 → 93 or 94 months