The Multiplication Principle

Example: Say a yogurt shop has three flavors (C, V and S) and two sizes, (L and M). How many different yogurts can be ordered?

We make a TREE DIAGRAM to help organize our choices.

What if there were 5 different toppings?

Use the *Multiplication Principle*:

MULTIPLICATION PRINCIPLE:

Suppose a task T_1 can be done N_1 ways and a task T_2 can be done N_2 ways and so on until task T_n can be done N_n ways. Then the number of ways of performing the tasks $T_1, T_2, \ldots T_n$ is given by the product

$$N_1 * N_2 \ldots * N_n$$

So if we have three kinds of yogurt with two sizes and five toppings the number of ways a yogurt can be chosen is

$$3 \quad * \quad 2 \quad * \quad 5 \quad = 30$$

Example - Older Texas plates have 3 letters followed by two numbers and a letter. How many different plates are possible?

Answer – Draw a blank for each choice

____ ____ ____ ____ ____ ____

How many ways if no repeats are allowed?

____ ____ ____ ____ ____ ____

Permutations

You want to arrange 10 students in a row of chairs.

FACTORIALS:

n! = n (n-1)(n-2) . . . 3 * 2 * 1 and 0!=1

Example - We have 10 students again, but only 4 chairs. How many ways can we seat the 10 students if we want to seat 4 of them in the chairs?

PERMUTATIONS. If we have a set of n elements and we want to take r of them in an arrangement, we say the number of permutations of n things taken r at a time is P(n, r).

In our second example we had 10 students taken 4 at a time, so

In general,

$$P(n,r) = \frac{n!}{(n-r)!}$$

in your calculator it is `nPr`

Example - How many ways can gold, silver and bronze medals be awarded in a race of 12 people?

Answer - We have 12 objects arranged 3 at time or

Same answer as the multiplication principle, What if the n objects contain some that are identical?

Count only the DISTINGUISHABLE permutations (the ones that look different).

If there are n_1 items of type 1 and n_2 items of type 2 and ... n_r items of type r, then the number of distinguishable permutations of the $n=n_1 + n_2 + ... + n_r$ items is

Example - we have 4 red marbles, 3 green marbles and one blue marble. How many distinguishable permutations of the 8 marbles are there?

Combinations

How many ways can we choose a *group* of 4 students the 10 students?

How many ways we can arrange them?

How many ways can the group be re-arranged?

So the number of ways a group of 4 can be chosen is

This is the number of *combinations* of 10 items taken 4 at a time.

$$C(n,r) = \frac{n!}{(n-r)!r!}$$

Example - Suppose a group of 7 workers decides to send a delegation of 2 to their supervisor.

How many delegations are possible?

If there are two women and 5 men in the group of workers, how many delegations will include at least one woman?

Exactly one woman with exactly one man or exactly two women.

How many ways to have two women on the delegation?

How many ways to have one woman and one man on the delegation?

Example - A bag contains 5 blue, 1 green and 3 orange jelly beans. You choose 3 at random. How many samples are possible in which

a) the jelly beans are all blue?

b) the jelly beans are all green?

c) the jelly beans are all orange?

d) there are 2 blue and 1 orange?

e) How many ways to choose 3 jelly beans?

f) How many ways to choose no blue?

g) How many ways to choose at least one blue?

A social security number has 9 digits. How many are possible if the first three digits are not zero?

How many ways can a hand of 6 clubs be chosen from a standard deck?

From 10 names on a ballot, how many ways can 4 be elected to a committee if each has a different responsibility?

A sales person has 6 prospects. How many ways can she arrange her schedule to see all 6?

You have a group of 13 different books. Three are math books, four are chemistry and six are history books. How many different arrangements are possible if books of the same type are kept together?

Use of Counting Techniques in Probability

Let S be a uniform sample space and E be any event in S. Then

$$P(E) = \frac{\text{number of outcomes in event E}}{\text{number of outcomes in the Sample Space}} = \frac{n(E)}{n(S)}$$

Example - suppose we have a jar with 8 blue and 6 green marbles. What is the probability that in a sample of 2, both will be blue?

What is the probability there is at least one blue marble?

Find the probability distribution table for the number of blue marbles in the sample of 2 marbles:

A lottery chooses 6 of 54 numbers be chosen and the order doesn't matter. What is the probability of choosing no winning numbers?

What is the probability of choosing at least 3 winning numbers?

Example - a stack of 100 copies has 3 defective papers. What is the probability that in a sample of 10 there will be no defective papers?

A bag contains 12 oranges and 2 are rotten. Take a sample of 3. Find the probability distribution table for the number of rotten oranges in the sample

Example - A student takes a true/false test with 5 questions by guessing (choose answer at random). Write a probability distribution table for the number of correct answers.

The Binomial Distribution

In a Bernoulli trial we have the following:

- The same experiment repeated several times.
- The only possible outcomes of these experiments are success or failure.
- The repeated trials are independent so the probability of success remains the same for each trial.

BINOMIAL PROBABILITY:
If p is the probability of success in a single trial of a binomial (Bernoulli) experiment, the probability of x successes and n-x failures in n independent repeated trials of the same experiment is

$$P(x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

Example - the first half of July was very dry in college station. If each day there was a 20% chance of rain, what is the probability of no rain in the first 15 days in July?

DEFINE SUCCESS:

n = number of trials =

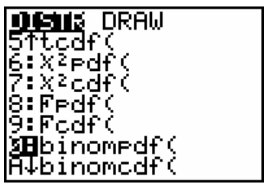
x = number of successes =

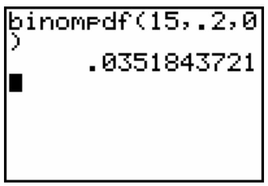
n-x = number of failures =

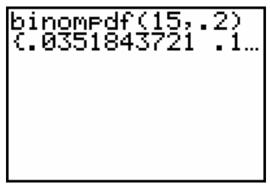
p = probability of success =

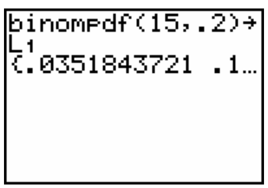
1-p = probability of failure = q =

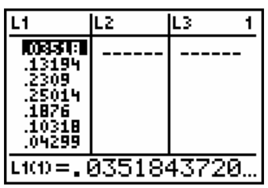
binompdf(n, p, x) on the calculator





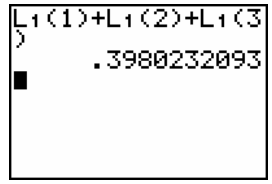




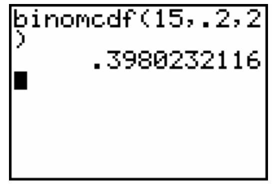


What is the probability of at most 2 rain days?

x = number of successes =



binomcdf(n, p, x) will give you the sum of the probabilities from 0 to x.



For a binomial probability question you must do the following:

- Decide that it is a Bernoulli trial.
- Define what success is.
- Find the number of times the experiment is done, n.
- Find the probability of success, p.
- Determine the number of successes you need to find, x.

Example - A new drug being tested causes a serious side effect in 5 out of 100 patients. What is the probability that in a sample of 10 patients none get the side effect from taking the drug?

define success = no side effect,

n = number of trials =

x = number of successes =

p = probability of success =

define success = side effect,

n = number of trials =

x = number of successes =

p = probability of success =