Formulating Linear Systems

You have a total of $86 in one-, five- and ten-dollar bills. There are 27 bills. You have twice as many ones as fives. How many of each type of bill do you have?

1. Underline the sentence that begins with “How many”. What are the variables?
   \[ x = \text{the ________ of one-dollar bills} \]
   \[ y = \text{the ________ of five-dollar bills} \]
   \[ z = \text{________________________________________________} \]

2. Write an equation for the statement “There are 27 bills.”
   \[ \text{________________________} \]

3. 
   a. If you have 3 five-dollar bills, how much money do you have? $_______
   b. If you have \( y \) five-dollar bills, how much money do you have? $_______
   c. Write an equation for the statement “You have a total of $86 in one-, five-, and ten-dollar bills.”
   \[ \text{________________________} \]

4. Make a drawing or a table to represent the statement “You have twice as many ones as fives.” 
   (See Part I, 5 for examples.)

5. Which equation represents the ratio of one-dollar bills to five-dollar bills?
   \[ x = 2y \quad \text{or} \quad y = 2x \]

6. Write the complete system of equations for this problem.
Zelda has $8,900 to invest. She decides to invest all of her money in three different funds. The PX Company costs $50 per share and pays dividends of $1.00 per share each year. The NY Company costs $90 per share and pays dividends of $2.00 per share each year. The LZ Company costs $40 per share and pays dividends of $1.60 per share per year. Zelda wants to invest half as much money in the LZ Company as in the NY Company and wants to earn $222 in dividends per year. How many shares of each company should Zelda buy to meet her goal?

1. What are the variables?

\[ x = \text{________________________________________} \]

\[ y = \text{________________________________________} \]

\[ z = \text{________________________________________} \]

2. “The PX Company costs $50 per share and pays dividends of $1.00 per share each year.”

   a. If you buy 5 shares of the PX company,
      
      How much does it cost? $_____  How much do you earn in dividends? $_____  

   b. If you buy \( x \) shares in the PX Company,
      
      How much does it cost? $_____  How much do you earn in dividends? $_____  

3. “The NY Company costs $90 per share and pays dividends of $2.00 per share each year.”

   a. If you buy 10 shares of the NY company,
      
      How much does it cost? $_____  How much do you earn in dividends? $_____  

   b. If you buy \( y \) shares in the NY Company,
      
      How much does it cost? $_____  How much do you earn in dividends? $_____  

4. “The LZ Company costs $40 per share and pays $1.60 per share per year in dividends.”

   a. If you buy 4 shares of the LZ Company,
      
      How much does it cost? $_____  How much do you earn in dividends? $_____  

   b. If you buy \( z \) shares in the LZ Company,
      
      How much does it cost? $_____  How much do you earn in dividends? $_____
5. Write an equation for the statement “Zelda has $8,900 to invest.”
   *Hint*: The amount of money invested is the same as the cost of purchasing stock.

6. Write an equation for the statement “Zelda wants to earn $222 in dividends per year.”

7. Which picture represents the statement “Zelda wants to invest half as much money in the LZ Company as in the NY Company.”

   ![Diagram A](image1.png)  ![Diagram B](image2.png)

8. Looking at your results from 3b., how much money does Zelda invest in the NY Company? _______

9. Looking at your results from 4b., how much money does Zelda invest in the LZ Company? _______

10. Using your answers from 8. and 9., what is the ratio equation for the statement “Zelda wants to invest half as much money in the LZ Company as in the NY Company.”?

11. Write the complete system of equations for this problem.
There are many ways to solve systems of linear equations; one way involves the use of matrices. A system of linear equations can be abbreviated in an **augmented matrix**. The key to converting a system of equations into an augmented matrix is the **alignment of variables**. After alignment, the coefficients of the variables and constants are placed into a matrix, separated by a vertical bar where the “=” from the equations was located. The system of equations can then be solved working only with the augmented matrix.

For example,

<table>
<thead>
<tr>
<th>Equations</th>
<th>Augmented Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>3x + 15y = 27</td>
<td>[ \begin{array}{rrr} 3 &amp; 15 &amp; 27 \ -2 &amp; 3 &amp; 8 \end{array} ]</td>
</tr>
<tr>
<td>−2x + 3y = 8</td>
<td></td>
</tr>
</tbody>
</table>

Notice here that, in the equations, the \( x \)-terms and \( y \)-terms are vertically aligned, as are the constants to the right of the “=".

**1.** In the number of apartments problem we created the following system of linear equations:

\[
\begin{align*}
x + y + z &= 192 \\
y + z &= x \\
x &= 3z
\end{align*}
\]

(1)

where \( x \) = number of small apartments, \( y \) = number of large apartments, and \( z \) = number of luxury apartments.

In this example, not all of the variables are on the left side of the equal signs with the constants on the right. Begin by first rearranging system (1) so that all the variables are aligned to the left of the “=” and constants are aligned to the right. Then write the system as an augmented matrix. If an equation does not contain a variable, the coefficient of this variable is 0.

\[
\begin{align*}
x + y + z &= 192 \\
y + z &= x \\
x &= 3z
\end{align*}
\]  

\[
\begin{array}{rrr|rr}
1 & 1 & 1 & \_ & \_ \\
0 & -1 & 1 & \_ & \_ \\
1 & 0 & 0 & \_ & \_ \\
\end{array}
\]

2. Write an augmented matrix for the system of equations found for the number of each type of bill.

3. Write an augmented matrix for the system of equations found for the number of shares Zelda should buy.