Matrix Multiplication

To perform matrix multiplication, always use *rows of the first* (left) matrix and *columns of the second* (right) matrix. Start at the left of a row and the top of a column and move across the row as you move down the column, "multiplying and adding."

Example: To calculate the element in Row 1, Column 2 of the product matrix, *AB* below, use Row 1 of matrix *A* and Column 2 of matrix *B*.

$$C = AB = \begin{bmatrix} \boxed{3} & \boxed{1} \\ 2 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & \boxed{0} & 2 \\ -1 & \boxed{2} & 3 \end{bmatrix} = \begin{bmatrix} x & \boxed{c_{12}} & x \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} = \begin{bmatrix} x & \boxed{3(0)+1(2)} & x \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} = \begin{bmatrix} x & \boxed{2} & x \\ x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

2. a. In order to obtain the value for the entry c_{23}, use Row 2 of matrix *A* and Column 3 of matrix *B*

$c_{23} =$ _____

$$C = AB = \begin{bmatrix} 3 & 1 \\ \boxed{2} & \boxed{1} \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \boxed{2} \\ -1 & 2 & \boxed{3} \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & \boxed{c_{23}} \\ x & x & x \\ x & x & x \end{bmatrix}$$

b. In order to obtain the value for the entry c_{41}, use Row ___ of matrix ___ and Column ___ of matrix ____.

$c_{41} =$ _____

$$C = AB = \begin{bmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \\ \boxed{c_{41}} & x & x \end{bmatrix}$$

3. a. Multiply the matrices.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} (1)(a)+(2)(c) & (\_\_\_)(\_\_\_)+(\_\_\_)(\_\_\_) \\ (\_\_\_)(\_\_\_)+(\_\_\_)(\_\_\_) & (\_\_\_)(\_\_\_)+(\_\_\_)(\_\_\_) \end{bmatrix} = \begin{bmatrix} a+2c & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ \end{bmatrix}$$

b. Multiply the matrices.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \_\_\_\_\_ & \_\_\_\_\_ \\ \_\_\_\_\_ & \_\_\_\_\_ \end{bmatrix}$$

Are the products in **a.** and **b.** equal? _____

In general, $AB \neq BA$ where *A* and *B* are both matrices.

If a matrix product must make sense, as when multiplying matrices where the entries have meaning the **inner dimensions must match in number and in meaning.**

If the product is meaningful, **the outer dimensions determine the meaning of the entries in the product matrix.**

Katie has earned enough from her flower shop to open a second flower shop in nearby Foundly. In one week she sold the following numbers of flowers:

	Roses	Carnations	Daisies	Orchids
Dexton	23	54	37	12
Foundly	18	46	41	10

4. Let matrix *C* represent the cost per stem of each flower, and let matrix *N* represent the number of each stem sold at each flower shop. Determine if the product *NC* is meaningful and if so, calculate *NC*.

$$N = \begin{array}{c} \\ D \\ F \end{array}\begin{array}{c} \begin{array}{cccc} r & c & d & o \end{array} \\ \begin{bmatrix} 23 & 54 & 37 & 12 \\ 18 & 46 & 41 & 10 \end{bmatrix} \end{array} \qquad C = \begin{array}{c} \\ r \\ c \\ d \\ o \end{array}\begin{array}{c} \begin{array}{c} \$ \end{array} \\ \begin{bmatrix} 3 \\ 1.50 \\ 2 \\ 2.50 \end{bmatrix} \end{array}$$

a. The 4 columns of *N* represent _____ and the 4 rows of *C* represent _____.
b. Do these inner dimensions match in meaning? _____ Therefore the produce IS or IS NOT meaningful. _____
c. The row(s) of the product matrix represent _____ and the column(s) represent _____.

d. $$NC = \begin{array}{c} \\ D \\ F \end{array}\begin{array}{c} \begin{array}{c} \$ \end{array} \\ \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \end{bmatrix} \end{array}$$

e. What is the meaning of the entry in the first row of the product *NC*? _____

What is the meaning of the entry in the second row of the product *NC*? _____

5. Now let Matrix *X* represents the number of units of wood (W), paint (P), and hardware (H) needed to make a chair (C), a table (T), and a desk (D). Matrix *Y* represents the cost for one unit of each of the necessary materials at plants located in Michigan (M), Florida (F), and Nevada (N).

$$X = \begin{array}{c} \\ W \\ P \\ H \end{array}\begin{array}{c} \begin{array}{ccc} C & T & D \end{array} \\ \begin{bmatrix} 4 & 1 & 2 \\ 12 & 3 & 6 \\ 7 & 5 & 14 \end{bmatrix} \end{array} \qquad Y = \begin{array}{c} \\ M \\ F \\ N \end{array}\begin{array}{c} \begin{array}{ccc} W & P & H \end{array} \\ \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 2 & 4 \end{bmatrix} \end{array}$$

Hint: Since each matrix has meaning, only one of the products *XY* or *YX* will be meaningful.

a. Calculate the value and determine the meaning (if any) of the entry in row 1, column 1 of the product matrix *XY*. In the spaces below put both the numerical value and units of each entry. At the end, say what the sum of the products is equal to and what it means. If it is not meaningful, enter "meaningless".

$(XY)_{11}$ = 4 units of wood to make a chair * $1 per unit of wood in Michigan + _____ * _____ + _____ * _____
= _____ which is _____

b. Calculate the value and determine the meaning (if any) of the entry in row 1, column 1 of the product matrix *YX*.

$(YX)_{11}$ = $1 per unit of wood in Michigan * 4 units of wood to make a chair

+ _____ * _____

+ _____ * _____

= _____ which is _____

c. Which of the two products are meaningful? *XY* or *YX*? ____

What does the meaningful product represent? _____

_____