Setting Up Linear Programming Questions

An apartment complex is being developed that will offer renters the choice of a one- or two-bedroom apartment. Based upon market research, the property owner has decided that half of the number of two-bedroom apartments is to be less than or equal to the number of one-bedroom apartments. To ensure that the complex has enough units to meet demand, at least 50 one-bedroom units and 30 two-bedroom units must be built. Each one-bedroom unit requires 4 panes of glass for windows, while each two-bedroom unit requires 8 panes. Due to pressure from the supplier, at least 640 panes of glass must be used. The cost to build a one-bedroom apartment is $67,000 and the cost to build a two-bedroom apartment is $81,000. How many of units of each type that must be constructed in order to minimize the cost of building all units.

Define Variables:
1. Note the sentence that begins with "How many units of"

2. *Define* the variables that will give us the number of each type of unit and also the variable which will be used for the objective function.

Let x = the # of one-bedroom apartments
Let y = the " " 2 " "
* Let C = the cost in $ to build apts * *

Create the Objective Function:
3. Is the owner maximizing or minimizing the total cost?
(A) Maximizing
(B) Minimizing
(C) Not clear

4. Write an expression for the total cost of constructing *all* one-bedroom units in the complex. $67000x$

5. Write an expression for the total cost of building *all* two-bedroom units in the complex. $81000y$

6. Write the expression for the total cost of producing all one- and two-bedroom apartments in the complex to complete the objective function: C = $67000x + 81000y$

Be sure to write C= when you are on your own !!

Create the Constraints:

$\boxed{x}$ $\boxed{y}$ $\Rightarrow \frac{1}{2}y = x$

7. "Based upon market research, the property owner has decided that half of the number of two-bedroom apartments is to be less than or equal to the number of one-bedroom apartments."

(A) ~~$y \le \frac{1}{2}x$~~ (B) ~~$y \ge \frac{1}{2}x$~~ (C) $\frac{1}{2}y \ge x$ (D) $\frac{1}{2}y \le x$ (E) None of these

Write a linear inequality to represent the quoted statement. ________

8. "To ensure that the complex has enough units to meet demand, at least 50 one-bedroom units and 30 two-bedroom units must be built."

a Write *two* linear inequalities to represent the quoted statement.

$x \ge 50$, $y \ge 30$

9. "Due to pressure from the supplier, at least 640 panes of glass must be used."

a. What is the total number of panes needed for *all* one-bedroom units in the complex? $4x$

b. What is the total number of panes needed for *all* two-bedroom units in the complex? $8y$

c. Write a linear inequality to represent the quoted statement.

$4x + 8y$ $\ge$ 640

10. Since the variables in this problem represent real-world quantities, write two additional inequalities that reflect this condition.
Additional Constraint 1: $x \geq 0$ Additional Constraint 2: $y \geq 0$

In this particular problem, these additional constraints have already been satisfied by the constraints in **8.**, and, therefore, it is optional to write these additional constraints in the final set-up of the problem.

Put It All Together (Set Up the Linear Programming Problem):

11. Complete the information below to state the linear programming problem in terms of its objective and constraints.

x = # of 1 bed apt
y = " " 2 " "
C = cost in $ for apt

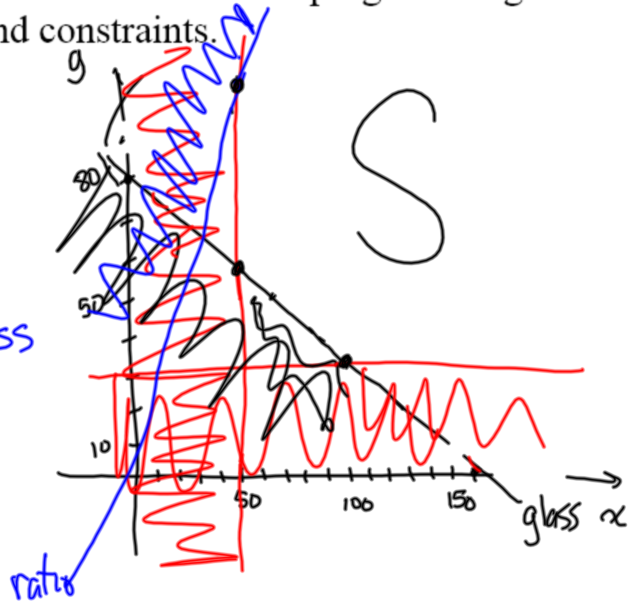
Min $C = 67000x + 81000y$

SUBJECT TO

(0, 80) and (160, 0) $4x + 8y \geq 640$ panes of glass

(0,0) and (1,2) $\frac{1}{2}y \leq x$ ratio

$x \geq 50 \quad y \geq 30$

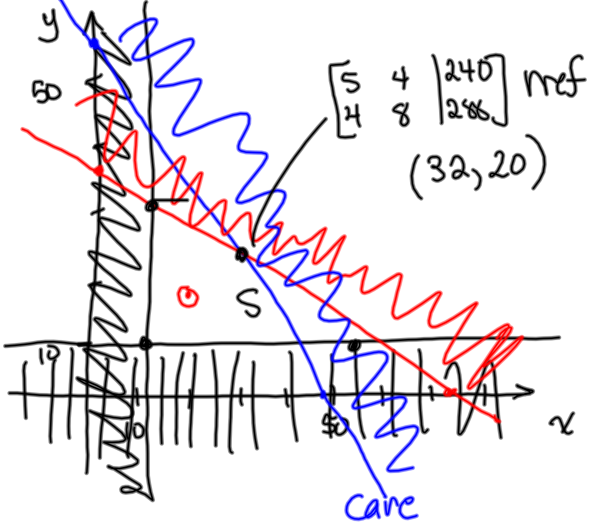


Set up the following linear programming problem.

12. A pet store sells large angelfish for \$20 each and silver dollarfish for \$16 each. Each angelfish requires 5 hours of care and 4 ounces of flake food to reach full size, and each silver dollarfish requires 4 hours of care and 8 ounces of flake food to reach full size. The pet store has 288 ounces of food and 240 hours of care available to raise the two kinds of fish. The store wants to sell a minimum of 12 angelfish and 10 silver dollarfish. If the store wishes to maximize the revenue obtained from selling angelfish and silver dollarfish, how many of each kind of fish should be raised to full size?

x = # of angel fish (AF)
y = # of silver dollar fish (SD)
R = revenue from fish in \$

Max $R = 20x + 16y$
SUBJECT TO
$5x + 4y \le 240$ hrs of care
$4x + 8y \le 288$ oz of food
$x \ge 12$ $y \ge 10$



$\left[\begin{array}{cc|c} 5 & 4 & 240 \\ 4 & 8 & 288 \end{array}\right]$ rref

$(32, 20)$

How much food is left over if $x=20$ and $y=20$? 48 oz

Have we maximized revenue? NO

Do we have enough resources to have 20 AF and 20 SDF?
(A) Yes (B) No, not enough care hours (C) No, not enough food
(D) No, not enough of both

Solving Linear Programming Problems

Every linear programming problem has a feasible region associated with the constraints of the problem. These feasible regions may be bounded, unbounded or the empty set.

To find the solution (that is, where the maximum or minimum value occurs), we will use the two theorems below.

Theorem 1 If a linear programming problem has a solution, then it must occur at a vertex, or corner point, of the feasible set S, associated with the problem. Furthermore, if the objective function P is optimized at two adjacent vertices of S, then it is optimized at every point on the line segment joining these vertices, in which case there are infinitely many solutions to the problem.

Theorem 2 Suppose we are given a linear programming problem with a feasible set S and an objective function $P = ax + by$.

- **Case 1** If S is bounded, then P has both a maximum and a minimum value on S.
- **Case 2** If S is unbounded and both a and b are nonnegative, then P has a minimum value on S provided that the constraints defining S include the inequalities $x \geq 0$ and $y \geq 0$.
- **Case 3** If S is the empty set, then the linear programming problem has no solution; that is, P has neither a maximum nor a minimum value.

Example

A craftsman has 150 units of wood, 90 units of glue and 150 units of paint. A small picture frame requires 1 unit of wood, 1 unit of glue and 2 units of paint while a large picture frame requires 5, 2 and 1 respectively. If a small frame sells for \$175 and a large frame for \$400, how many of each should be made to maximize the revenue? Are there any leftovers when the revenue is maximized?

What is the maximum revenue?