

Clicker question: Referring to the pet store's sale of angelfish and silver dollar fish, how many ways can they have a maximum profit of \$960?

- (A) 1 way (B) 2 ways (C) 3 ways (D) More than 3, but not infinite
(E) There are an infinite number of ways to maximize profits

Quiz 4 is available to be picked up at the front of the room

(I have exam 1, if you have not already picked it up)

Please do not turn in the activity if you used your clicker

😊 Print your name neatly so the grader and I can read it 😊

★ Be sure to double check that the grades are entered into WebAssign correctly. ★

The following clickers have been to class, but are not registered:

#8374C433 #84FC0971 #8DF5F48C #9B191F9D

Please see me before or after class if your clicker is on this list.

*TYPO ⇒ medium
not small*

A *set* is a collection of objects.

The objects in a set are the *elements* or *members* of the set.

→ Always enclose the elements of a set in curly brackets.

A set with the numbers $-1, 1, 0$ would be written as $\{-1, 1, 0\}$

$-1 \in \{-1, 1, 0\}$ where \in is read "is an element of"

Define $S = \{-1, 1, 0\}$ $-1 \in S$ $n(S) = 3$

More notation:

- 0 is the symbol for the real number zero
- $\{0\}$ is a set with one element, the real number zero
- \emptyset is a set with zero elements, the empty set. Alternative is $\{\}$.
- $\{\emptyset\}$ is a set with one element, the symbol for the empty set.

Two sets are *equal* ($=$) if they contain exactly the same elements (order doesn't matter).

$$\{1, 2, 3\} = \{3, 2, 1\} \text{ etc}$$

They are *not equal* (\neq) if they don't contain the same elements.

$$\{1, 2, 3\} \neq \{1, 2, 4\}$$

Set builder notation: Describe the set in terms of its properties,

$$E = \{x \mid x \text{ is a positive even integer less than } 17\}$$

Roster notation: List the elements of the set.

$$E = \{2, 4, 6, 8, 10, 12, 14, 16\} \text{ or } \{2, 4, 6, \dots, 16\}$$

Subset:

Set B is a subset of set A (written $B \subseteq A$) if every element in B is in A .

$$F = \{x \mid x \text{ is a positive multiple of 4 less than } 17\} = \{4, 8, 12, 16\}$$

$$F \subseteq E \quad \text{also} \quad F \subseteq F \quad \text{and} \quad E \subseteq E$$

Proper Subset:

Set B is a proper subset of set A (written $B \subset A$) if $B \subseteq A$ and $A \neq B$.

$$F \subset E$$

Universal set:

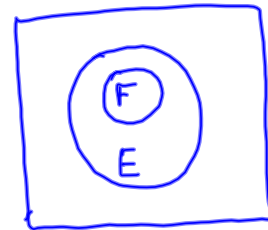
The set from which all the member of other sets will be drawn. Called U .

$$U = \{x \mid x \text{ is a positive integer less than } 17\}$$

$$= \{1, 2, 3, \dots, 16\}$$

Venn Diagram notation:

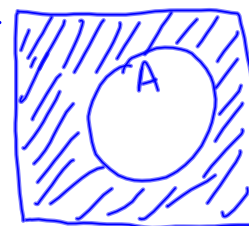
- A rectangle represents the universal set
- Circles are sets in the universal set.



Example: Show the relationship between E and F (defined above) in a Venn diagram.

Given a set A and a universal set U , the elements that are in U and are NOT in A is called the **complement** of A or A^c .

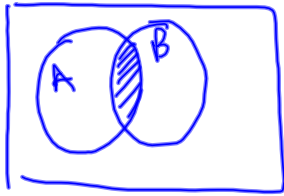
Shade A^c



Example: From the last example, E is the set of positive even integers less than 17, what is E^c in roster notation?

$$E^c = \{1, 3, 5, \dots, 15\}$$

Those elements that belong to both A and B are in the intersection of A and B , $A \cap B$.

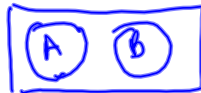


$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Example: Let $U = \{x \mid x \text{ is a card in a standard deck of 52 playing cards}\}$
 $R = \{x \mid x \text{ is a red card}\}$ and $Q = \{x \mid x \text{ is a queen}\}$

Find $R \cap Q$ in roster notation. $R \cap Q = \{Q \heartsuit, Q \diamondsuit\} = \{Q_H, Q_D\}$

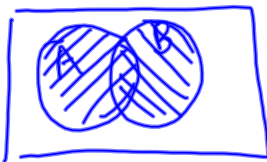
If two sets have no elements in common, that is $A \cap B = \emptyset$, then the sets are *disjoint*.



Example: If $K = \{x \mid x \text{ is a king}\}$, find $K \cap Q$ in roster notation.

$$K \cap Q = \emptyset \quad n(K \cap Q) = 0$$

Those elements that belong to A or B are in the union, $A \cup B$.



$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

★ Note: this is the *inclusive or*, not the exclusive or ★

Example Let $U = \{x \mid x \text{ is a card in a standard deck of 52 playing cards}\}$
 $H = \{x \mid x \text{ is a heart card}\}$ $Q = \{x \mid x \text{ is a queen}\}$

Find $H \cup Q$ in roster notation.

$$H \cup Q = \{A_H, 2_H, 3_H, \dots, J_H, Q_H, K_H, Q_D, Q_S, Q_C\}$$

$$n(H \cup Q) = 16$$

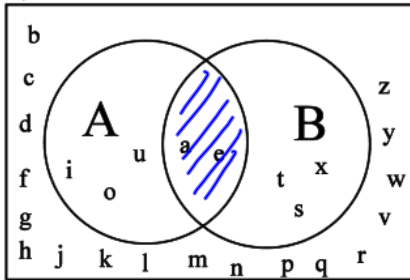
Example Let $U = \{x|x \text{ is a letter in the English alphabet}\} = \{a, b, c, \dots, z\}$

$A = \{x|x \text{ is a vowel}\} = \{a, e, i, o, u\}$

$B = \{x|x \text{ is a letter in the word texas}\} = \{t, e, x, a, s\}$

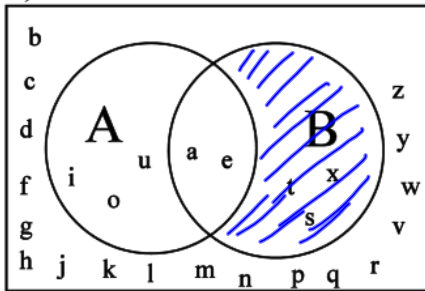
Find the following sets in roster notation.

a) What is $A \cap B$?



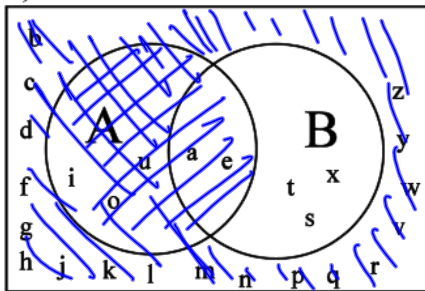
$$A \cap B = \{a, e\}$$

b) What is $A^c \cap B$?



$$A^c \cap B = \{t, x, s\}$$

c) What is $A \cup B^c$?



$$= \{a, b, c, \dots, r, u, v, w, y, z\}$$

SETS

Use the following sets to answer the true/false questions.

When finding complements of sets, assume sets A, B, C, D, and E are subsets of the universal set U1 and sets F, G, and H are subsets of the universal set U2.

Otherwise, just view sets A-H as a group of sets in one big universal set.

For the clicker answers, use A if the statement is TRUE and B if FALSE.

$U1 = \{x|x \text{ is a letter in the alphabet}\}$

$A = \{a, b, c\}$ $B = \{a, e, i, o, u\}$ $C = \{d, e, g, h, z\}$

$D = \{b, c, a\}$ $E = \{w, x, y, z\}$

$U2 = \{0, 1, 2, \dots, 10\}$

$F = \{0, 2, 4, 6, 8\}$ $G = \{1, 2, 3, 4, 5\}$ $H = \{1, 2, 7, 8\}$

(a) $A = D$ is T

(b) $\emptyset \in F$ is F $(\emptyset \in F \text{ is true})$ $(\emptyset \subset F \text{ is true})$ $F^c = \{1, 3, 5, 7, 9, 10\}$
 (Note: 10 is in U2)

(c) $B \subset U1$ is T

(d) $F^c = \{x|x \text{ is an odd integer between } 0 \text{ and } 10\}$ is F
 (Note: $\{1, 3, 5, 7, 9\}$)

(e) $B^c \cap C = \{d, g, h, z\}$ is T

(f) $G^c \cup F = \{0, 2, 4, 6, 7, 8, 9, 10\}$ is T

(g) $G \subseteq U2$ is T

(h) $\emptyset \subset E$ is T

(i) E has 16 proper subsets is F a set with n elements has 2^n subsets

(j) $A \cup (B \cap E)^c = U1$ is T $2^4 = 16$, $16 - 1 =$ proper subsets

(k) $\{3\} \in G$ is F $B \cap E = \emptyset$, $(B \cap E)^c = U1$
 $A \cup U1 = ?$

(l) $2 \in F$ is T

(m) $\emptyset = \{\emptyset\}$ is F

$3 \in G$ is true
 $\{3\} \subseteq G$ is true

(n) D has 8 subsets is T

(o) $A \subseteq D$ is T

(p) $\{w, x\} \subseteq E$ is T
 $n(C) = 5$, $n(C^c) = ?$ 21

A set with n elements has 2^n subsets
↓

(q) C^c has 2,097,152 subsets is 2^{21} T

PICK UP QUIZ 5
How many statements are false, just counting (a) - (z)?
(hint it's between 0 and 12)
(also, (u) is True)

(r) $A \cup B = \{a\}$ is F

(s) $H \subset G$ is F

(t) $n(B \cup F) = 10$ is T

(u) $\{a, b\} \notin A$ is T

$\{2, 8\}$

(v) $(H \cap F) \subseteq G^c$ is F $G^c = \{0, 6, 7, 8, 9, 10\}$

(w) $\{h, o, w, d, y\} \subset U_1$ is T

(x) $\emptyset = \{\}$ is T

(y) $n(C \cup H) = 0$ is F (9)

(z) $U_1 + U_2 = 37$ is F $n(U_1) + n(U_2) = 37$