Pick up your quiz up front

Clicker Question: How many answers are false on (n)-(z)?

note (u) is true. So is $a \in A \& \{a\} \subset A$ and $\{a\} \in A$ is false

Example: How many different 4 digit numbers can be made from the digits

1, 2, 3, 4, 5, 6, 7

a) If there are no restrictions? $7 \cdot 7 \cdot 7 \cdot 7 = 2401$ (1000's, 100's, 10's, 1's)

b) If the number must be even? $7 \cdot 7 \cdot 7 \cdot \underset{\text{even}}{3} = 1029$

c) If it is even and there are no repeats? $6 \cdot 5 \cdot 4 \cdot \underset{\text{even}}{3} = 360$

5556 or 5565 or 5655 or 6555 are OK. 5555 NOT

d) If four of the same digit is not allowed?

$7 \cdot 7 \cdot 7 \cdot 7 - 7 = 2394$ (no restr)

1111, 2222 .. 7777

Example: How many ways can 10 students be seated in a row of 10 chairs?

$10 \cdot 9 \cdot 8 \cdots 3 \cdot 2 \cdot 1 = 10! = 3628800$

10 MATH → PRB → 4:!

Example: How many ways can 4 of 10 students be seated in a row of 4 chairs?

$10 \cdot 9 \cdot 8 \cdot 7 = 5040$

Permutations: If we have a finite set of n elements and we want to place r of them in an arrangement, we say the number of permutations of n things arranged r at a time is $P(n, r)$.

Example How many ways can gold, silver and bronze medals be awarded in a race of 12 people?

$\underset{G}{12} \cdot \underset{S}{11} \cdot \underset{B}{10} = 1320 = P(12,3)$

Example How many ways can a group of 4 students be chosen from 10 students?

$\frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1} = C(10,4) = 10$ Math → PRB → nCr → 4 enter

$= 210$ = 10nCr4

Combinations: If we have a finite set of n elements and we want to take r of them in an group, we say the number of combinations of n things grouped r at a time is $C(n, r)$.

Example: How many ways can a hand of 6 clubs be chosen from a standard deck?

$\underset{6C}{C(13,6)} = 1716$

Example: From a group of 12 people, how many ways can a committee of 4 be formed if one person is the chair of the committee?

$\underset{\text{comm}}{C(12,4)} \cdot \underset{\text{ch}}{C(4,1)} = 1980 = \underset{\text{ch}}{C(12,1)} \; \underset{\text{rest of comm}}{C(11,3)}$

Example: A class of 12 students will divide into 3 teams of 4. How many ways can this be done?

$\underset{\text{team 1}}{C(12,4)} \cdot \underset{\text{team 2}}{C(8,4)} \cdot \underset{\text{team 3}}{C(4,4)} = 34,650$

Example: A bag contains 6 blue, 1 green and 3 pink jelly beans. You choose 3 at random. How many samples are possible in which

a) the jelly beans are all blue?

$\underset{3B}{C(6,3)} \cdot \underset{\text{AND } 0B^c}{C(4,0)} = 20$

b) the jelly beans are all green?

0

c) the jelly beans are all pink?

$1 = \underset{3P}{C(3,3)} \cdot \underset{0P^c}{C(7,0)}$

d) there are 2 blue and 1 pink?

$\underset{2B}{C(6,2)} \cdot \underset{1\text{ Pink}}{C(3,1)} \cdot \underset{0\text{ Green}}{C(1,0)} = 15 \cdot 3 \cdot 1 = 45$

e) How many ways to choose 3 jelly beans?

$\underset{\text{Pick 3}}{C(10,3)} = 120$

f) How many ways to choose no blue?

$\underset{0B}{C(6,0)} \cdot \underset{3B^c}{C(4,3)} = 4$

g) How many ways to choose at least one blue?

$\underset{1B}{C(6,1)} \cdot \underset{2B^c}{C(4,2)} + \underset{2B}{C(6,2)} \cdot \underset{1B^c}{C(4,1)} + \underset{3B}{C(6,3)} \cdot \underset{0B^c}{C(4,0)} = 116$

(OR) (OR) and

$120 - \underset{0\text{Blue}}{4}$

Example: A school is putting together a committee. The committee will have a chair and an assistant chair chosen from a group of 10 teachers, two parents chosen from a group of 15 parents and two students chosen from a group of 20 students. How many different committees are possible?

$\frac{10}{\text{ch}} \cdot \frac{9}{\text{AC}} \cdot \frac{C(15,2)}{\text{2 Parents}} \cdot \frac{C(20,2)}{\text{2 students}}$

Example: You are dealt a hand of four cards from a well-shuffled standard deck of 52 cards.

(a) How many ways can you be dealt at least 3 spades?

$\frac{C(13,3)}{3\,S} \cdot \frac{C(39,1)}{\text{and } 1\,SC} + \frac{C(13,4)}{4S} \cdot \frac{C(39,0)}{\text{and } 0\,SC} = 11869$ (OR)

(b) How many ways can you be dealt exactly two diamonds or exactly two clubs?

$E = 2\diamondsuit \quad n(E) = \frac{C(13,2)}{2D} \cdot \frac{C(39,2)}{2DC} = 57798$

$F = 2C \quad n(F) = \frac{C(13,2)}{2C} \cdot \frac{C(39,2)}{2C^C} = 57798$

$n(E \cap F) = \frac{C(13,2)}{2C} \cdot \frac{C(13,2)}{2D} = 6084$

$n(E \cup F) = n(E) + n(F) - n(E \cap F) = 57798 + 57798 - 6084$

$= 109,512$

Example: You have 2 different math books, 4 different history books and 5 different fiction books. How many ways can these books be arranged on a shelf if books of the same type are kept together?

$M_1 M_2 \quad H_1 H_2 H_3 H_4 \quad F_1 F_2 F_3 F_4 F_5$

$\frac{3 \cdot 2 \cdot 1}{\text{arr GP}} \cdot \frac{2 \cdot 1}{M} \cdot \frac{4 \cdot 3 \cdot 2 \cdot 1}{H} \cdot \frac{5!}{F} = 34,560$

Counting Activity – All questions will count down from 4 minutes

Problem 1: In how many ways can a committee with 12 members elect a chair, vice chair, and a subcommittee of 5 members if officers cannot serve on the subcommittee?

# of ways = 33,264 $= \underbrace{12}_{C} \cdot \underbrace{11}_{VC} \cdot \underbrace{C(10,5)}_{SUB}$

Problem 2: A box of 24 batteries contains 9 defective batteries. If you randomly select a sample of 6 batteries, how many samples have at least one defective battery?

# of ways = 129,591 $= \underbrace{C(24,6)}_{\text{all possible samples}} - \underbrace{C(9,0) \cdot C(15,6)}_{\text{0 Def and 6 GOOD from 9 from 15}}$

$\Rightarrow$ so 15 are "GOOD"

Problem 3: How many license plates with 3 letters and 3 digits can be formed if letters must be kept together, digits must be kept together, no letters can be repeated, and each of the 3 digits are odd?

# of ways = 3,900,000 $= \underbrace{8}_{\text{L or \#'s 1st}} \cdot \underbrace{26 \cdot 25 \cdot 24}_{\text{L (no repeats)}} \cdot \underbrace{5 \cdot 5 \cdot 5}_{\text{odd digits}}$

Problem 4: How many 4-digit PIN codes can be formed if zero cannot be the first digit, the last digit must be even, and no digits can be repeated?

# of ways = 2296 $= \underline{\quad}\,\underline{\quad}\,\underline{\quad}\,\underbrace{\underline{\quad}}_{\text{zero}} + \underline{\quad}\,\underline{\quad}\,\underline{\quad}\,\underbrace{\underline{\quad}}_{2,4,6 \text{ or } 8}$

Counting Activity – All questions will count down from 4 minutes

Problem 5: Bob and Jan go to the movies with 6 of their friends. They all sit together in the front row, which has exactly 8 chairs. How many different arrangements of these 8 people are possible if Bob and Jan must sit next to each other?

# of ways = 2 * 7!

Problem 6: A class is taking a 10-question multiple choice exam. The professor, in an effort to reward good attendance, has indicated that 3 questions have the answer A, 2 questions have the answer B, 1 question has the answer C, and 4 questions have the answer D. Using this information, how many ways can a student answer the exam?

# of ways = 12,600