Binomial Probability

For a probability experiment to be binomial, it must have all of the following properties:

- **The number of trials in the experiment is fixed. This value is called *n*.**
- **There are only two outcomes: Success and Failure.**
- **The probability of success in each trial is the same. This value is called *p*.**
- **The trials are independent of each other.**

Part I

1. In a large city 30% of the residents like broccoli (*B*) and the rest do not like broccoli (*N*). Three residents are chosen at random and asked if they like broccoli or not.

a. Are there a fixed number of trials in this experiment? ____ If yes, how many? n = ____

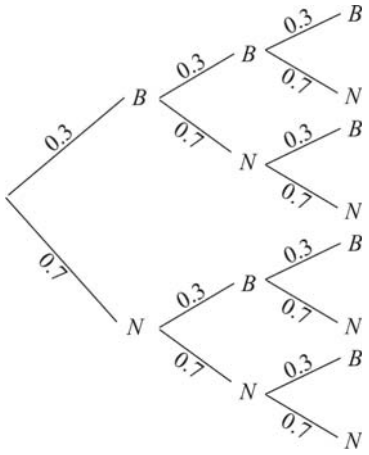
b. Are there only two outcomes in every trial? _____ If yes, what are they? _____ and _____

c. Is the probability of success in each trial the same? _____
If yes, a success is considered to be _____ and the probability of this occurring is p =_____

d. Does the outcome on one trial change the probabilities of the outcomes on other trials? _____
If no, each trial is said to be _____ of each other.

e. Is this a binomial experiment? _____

2. The tree diagram shows this experiment. How do the properties of binomial experiments relate to this tree diagram?



a. There are ____ "stages" in the tree, which correspond to _____

b. Each "node" has _____ branches that correspond to

c. The probabilities on the tree DO or DO NOT change (circle one) on each pair of branches throughout the tree, which means that _____

3. Let X = # of successes = # of residents who like broccoli.
Find the following probabilities using the tree diagram and the product rule.

a. Calculate the probability that 0 residents like broccoli, or $P(X = 0)$. _____

b. Calculate the probability that exactly 1 resident likes broccoli, i.e., 1 success. _____

c. Calculate the probability that exactly 2 residents like broccoli, i.e., 2 successes. _____

d. Calculate the probability that all 3 residents like broccoli, i.e., 3 successes. _____

4. Organize this information in a probability distribution table with X = # of residents who like broccoli.

X	0	1	2	3
$P(X)$	_____	_____	_____	_____

The probability of r successes in a binomial experiment with n trials and probability of success p is

$$P(X = r) = C(n,r)\,p^r(1-p)^{n-r}$$

5. On your calculator, enter the X values into list L1 and $P(X)$ into list L2. Do 1-var stats L1, L2.

Mean = μ = _____ Standard deviation = σ = _____

What is $n \times p$? _____ What is $\sqrt{n \times p \times (1-p)}$? _____

For BINOMIAL experiments, $\mu = n \times p$ and $\sigma = \sqrt{n \times p \times (1-p)}$

Part II

The binomial experiment in **Part I** had the following properties:
X = # of successes = # of residents who like broccoli, n = 3, p = 0.30 and r = 0, 1, 2, or 3.
With this information and your calculator, no tree or formula is needed to calculate probabilities.

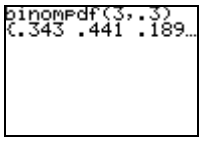
***P*(*X* = *r*) = binompdf(*n*, *p*, *r*)**

To access this command, go to DISTR (above the VARS button) and scroll down to find it as command 0:

Choose the value of r based on the specifics of the question you are asked.

1. What is the command to find the probability that exactly 2 residents like broccoli?
$P(X = 2)$ = binompdf(____, ____, ____) = _____ This is the same value as **Part I, 3c.**

If you simply enter n and p in binompdf, it will list the probabilities for every value of X (from $X = 0$ to $X = n$). Use the right and left arrows to scroll through the list.

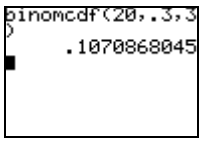


For the remaining questions, ***assume that 20 residents are surveyed*** from the same city as in **Part I**.

2. What is the probability that exactly 4 residents like broccoli?
- **a.** Success = likes broccoli, n = 20, p = _____, r = _____
- **b.** $P(X = 4)$ = binompdf(____, ____, ____) = _____

3. What is the probability that 3 or fewer residents like broccoli?
- **a.** r = desired # of successes = 0, ___ , ___, ___.
- **b.** $P(0 \le X \le 3) = P(X = 0)$ + _____ +_____ +_____
  = binompdf(____, ____, ____) + _____(____, ____, ____)
  + _____(____, ____, ____) + _____(____, ____, ____)
  = _____

The calculator command **binomcdf** sums the probabilities of 0 to *r* successes for you.



$$P(0 \le X \le r) = \text{binomcdf}(n, p, r)$$

The binomcdf command is found under the binompdf command in the DISTR menu.

c. Find the same probability found in **b** using the binomcdf command.
$P(0 \le X \le 3)$ = binomcdf(____, ____, ____) = _____

4. What is the probability that more than 8 residents like broccoli? r = 9, ____, … , ____

Although the desired number of successes does not begin at 0, the *complement* of the desired probability is r=0 to r=8 successes, which *can* be calculated with a binomcdf command.

$P(X > 8) = 1 - P(0 \le X \le 8) = 1 -$ binomcdf(____ , ____ , ____) = _____

5. What is the probability that more than 4, but fewer than 15 residents, like broccoli?
r = 5, ____, … , ____
This probability can be found by taking the probability of r=0 to r=14 successes and subtracting off the part of this probability that you do not want, r=0 to r=4 successes.

$P(5 \le X \le 14) = P(0 \le X \le 14) - P(0 \le X \le 4)$
= binomcdf(____, ____, ____) – _____(____, ____, ____) = _____

6. What is the probability that between 6 and 12 residents like broccoli?
Hint: The word **between** means *strictly* inside of the two endpoints.
r = ____, ____, … , ____
$P($____ $\le X \le$ ____$) = P(0 \le X \le$ _____$) - P(0 \le X \le$ _____$)$
= _____
= _____

7. What is the probability that between 2 and 5 people inclusive like broccoli?
Hint: The word **inclusive** means that you include the endpoints.
r = ____, ____, … , ____
$P($____ $\le X \le$ ____$) = P(0 \le X \le$ _____$) - P(0 \le X \le$ _____$)$
= _____
= _____

8. What is the mean and standard deviation in the number of people who like broccoli when 20 people are surveyed?

Mean = μ = _____ Standard deviation = σ = _____