

## 8.5 and 8.6: The Normal Distribution

Continuous random variables can take on any value.

Let  $t$  = time in seconds to run a race

Let  $w$  = weight of kitten in kg

Let  $L$  = length of a week-old bean plant

Let  $X$  = value where pointer lands.

$$0 \leq X < 1 \text{ and } P(0 \leq X < 1) = 1$$

What is  $P(X = \frac{1}{2})$ ?

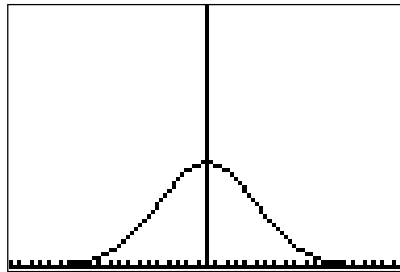
What is  $P(0 \leq X < \frac{1}{4})$ ?

What is  $P(0.75 \leq X \leq 0.80)$ ?

How can we represent this graphically?

When we graph the probability distribution for a continuous variable we find a probability density function.

Many natural and social phenomena produce a continuous distribution with a bell-shaped curve.

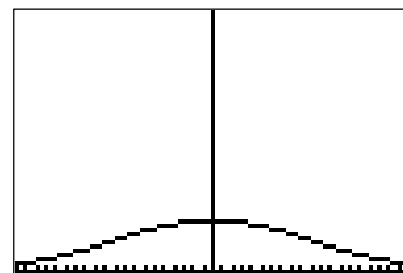
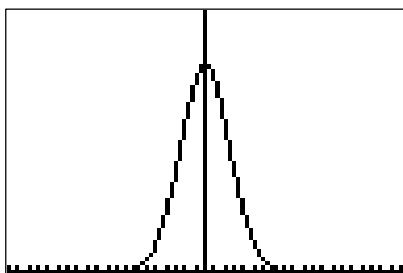


Every bell-shaped (NORMAL) curve has the following properties:

- Its peak occurs directly above the mean,  $\mu$
- The curve is symmetric about a vertical line through  $\mu$ . The curve never touches the x-axis. It extends indefinitely in both directions.
- The area between the curve and the x-axis is always 1 (total probability is 1).

The shape of the curve is completely determined by  $\mu$  and  $\sigma$ ,

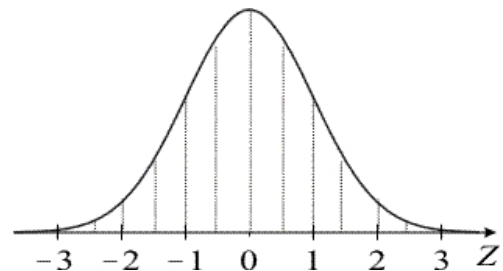
$$P(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



A continuous probability distribution is represented by a probability density function,  $f$ . The area between the graph of  $f$  and the  $x$ -axis from  $x = a$  to  $x = b$  gives the probability that the random variable  $X$  is between  $a$  and  $b$ .

The standard normal curve has  $\mu = 0$  and  $\sigma = 1$ .  $Z$  is used to represent the standard normal curve instead of  $X$  in this special case.

On the normal curve to the right, shade the area where  $-0.5 < Z < 0.5$ .

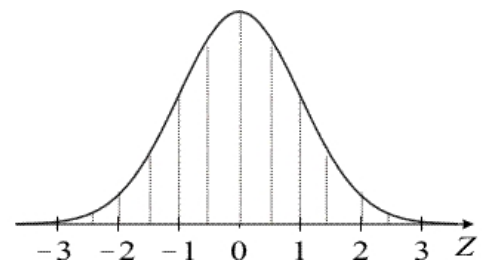


The normalcdf function on your calculator is used to evaluate probabilities involving normal curves.

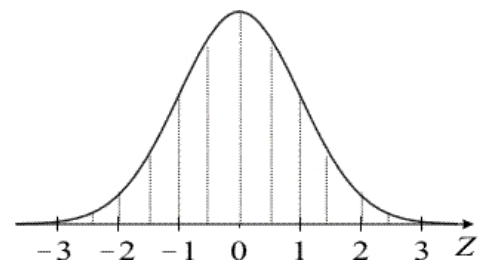
$$P(a < X < b) = \text{normalcdf}(a, b, \mu, \sigma)$$

$$P(-0.5 < Z < 0.5) = \text{normalcdf}(-0.5, 0.5, 0, 1) = \underline{\hspace{2cm}}$$

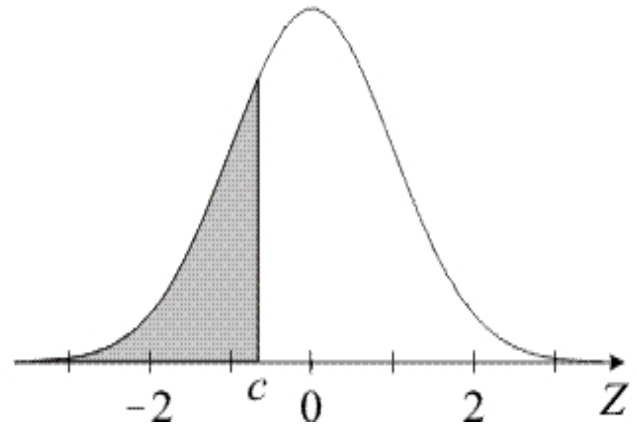
What is  $P(Z > 1.5)$ ?



What is  $P(Z < 2)$ ?



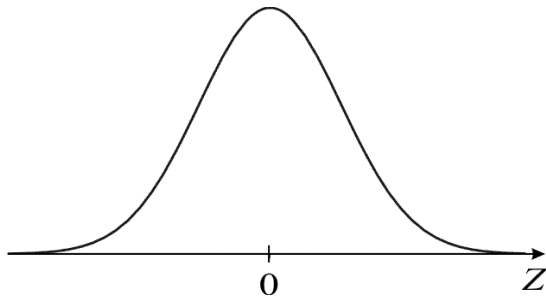
In the drawing on the right, the shaded region has an area of 0.25. To find the value of  $c$ , the calculator has a function to “invert the normal curve” called **invNorm**.



**invNorm(area to LEFT of unknown value,  $\mu$ ,  $\sigma$ )**

$$c = \underline{\hspace{2cm}}.$$

Find the value of  $b$  such that  $P(Z > b) = 0.10$ .



Find the value of  $a$  such that  $P(-a < Z < a) = 0.60$ .

