Some items of interest and reminders:

- Our last day of class is TUESDAY, May 5
- Office hours for May 4 - May 7 are TBA
- The review for the final is on WEDNESDAY, May 6 at 10A in CHEM 100
- The final exam is on THURSDAY, May 7 at 5:30 in BLOC 102 (I think)
- The final exam will be all multiple choice and have approximately the following
  - 25% from chapters 1 and 2
  - 25% from chapters 3 and 6
  - 25% from chapters 7 - 8.4
  - 25% from chapters 5 and 8.5-8.6
- The last homework is due on Tuesday, May 5
- The quiz on May 5 will be over 5.1 - 5.3
- There are a lot of study resources on the class webpage

Annuities and Amortization

The most common compounding frequencies are:

Compounding	Annually	Semi-Annually	Quarterly	Monthly	Weekly	Dail
# of times per year (m)	1	2	4	12	52	365

Part I

1. Ben would like to have \$60,000 available for college in 18 years.

a. If an account exists earning 7% annual interest compounded annually, how much should be deposited now to have the \$60,000 in 18 years?

N= 18
I= 7
*PV= ?
*PMT= 0
*FV = 60000
P/Y = 1

5.1

60000 − 17751.83

\$ 17,751.83

b. How much interest is earned? \$ 42,248.17

If you don't have all the money available now, you can save a bit each year to reach this goal. This is a type of annuity.

An **annuity** is an account into which *regular payments* are made. The annuities in this class are all CERTAIN and SIMPLE, meaning:

- The payments are made at fixed time intervals.
- The periodic payments are of equal size.
- The payments are made at the end of the interval.
- The interest is paid at the end of the interval.

c. Use the TVM Solver to find the amount of the annual payments needed to reach the goal of $60,000.

N = 18 I% = 7 PV = 0 PMT = ?

FV = 60000 P/Y = C/Y = 1

The amount of the annual payments is $1764.76.

d. In total, how much money was deposited into the account?

18*PMT = 18 x 1764.76 = 31,765.68

e.

$28,234.32 = 60000 − 31765.68

interest earned = final value of account − amount deposited

f. Why are the interest amounts in parts **b.** and **e.** different?

2. You are saving for retirement by depositing $400 per month in an account that earns 8% annual interest compounded monthly. Determine the total amount in the account and the total interest earned if this is done for 10 years, 30 years and 50 years.

$N = 10 \times 12 = 120$ $PMT = 400$
$I = 8$ $FV = ?$
$PV = 0$ $P/Y = 12$

$400 \times 120 = 48{,}000$ we deposited

After 10 years, $ 73,178.41 total and $ 25,178.41 interest

$N = 30 \times 12 = 360$ $PMT = 400$
$I = 8$ $FV = ?$
$PV = 0$ $P/Y = 12$

After 30 years, $ 596,143.78 total and $ 452,143.78 interest

$N = 50 \times 12 = 600$
deposited $600 \times 400 = \$240{,}000$

After 50 years, $ 3,172,690.99 total and $ 2,932,690.99 interest

Loans also deal with making payments. A bank loans you money and you make regular payments until you no longer have any of the bank's money. This means that the original value of the loan is the present value (PV) and the future value of the loan (FV) is 0.

3. You find a car advertised for no money down with payments of \$199 per month for 72 months. In the fine print you see that the interest is 9% annual interest compounded monthly on the unpaid balance.

a. What is the purchase price of the car? That is, how large was the loan you took out? \$ 11,039.89

N = 72 PMT = 199
I = 9 FV = 0
PV = ? P/Y = 12

b. How much money total will you pay out of pocket?

72*199 = 14,328

c. Interest Paid = 14,328 − 11,039.89 = \$3286.11
Amount of Money Repaid – Amount Borrowed =

4. Your goal is to retire when you become a multi-millionaire ($2,000,000) so you start saving $100 per week in an account that pays 10% annual interest compounded weekly. If you start at age 23, how old will you be when you retire?

N = ?
I = 10
PV = 0
PMT = 100
FV = 2,000,000
P/Y = 52

make one of these negative

→ N = 1913 WEEKS

$\frac{1913}{52} = 36.8$ year

$+ 23$

60 years old

5. A cello costs $998. You pay off the cello by making payments of $39.00 per month for 3 years.

a. What annual interest rate compounded monthly were you charged (rounded to 4 decimal places)?

N = 3 x 12 = 36
I = ?
PV = 998
PMT = 39
FV = 0
P/Y = 12

→ 23.7% int. rate

b. How much interest do you pay in total?

$36 \times 39 - 998 = 1404 - 998 = \$406$

Part II and quart. pmts are made

You take out a $1000 loan. The terms of the 1-year loan are 16% annual interest compounded quarterly on the unpaid balance.

N = 4 PMT = ?
I = 16 FV = 0
PV = 1000 P/Y = 4

1. How much are the quarterly payments? $ 275.49

4 x 275.49 − 1000 = 101.96

2. How much interest is paid in total? $ 101.96

3. If the interest was compounded annually, instead, and you just paid the entire loan off at the end of the year, how much would the total interest be?$ 160

I = Prt = 1000 (.16)(1)

4. Why are the answers in **2.** and **3.** different?

Each payment is broken down into two parts; the interest owed is paid first and then the rest of the payment goes towards paying off (paying down) the amount owed (principal).

The amount of interest paid is the largest in your first payment, while nearly all of the last payment goes towards the principal. **Even though the payment is the same each month, the distribution of interest and principal changes with every payment.**

5. What is the periodic, in this case quarterly, interest rate?

$\frac{.16}{4} = 0.04$

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6. How much of the first payment is interest and how much pays down the loan? *Hint*: Use the quarterly interest rate as a decimal.

1000	×	.04	=	40
mount owed		quarterly interest rate		quarterly interest owed

275.49	−	40	=	235.49
ayment amount		quarterly int owed		amt to principal

7. What is the outstanding principal after the first payment is made?
Hint: The current loan balance is $1000.

1000	−	235.49	=	764.51
urrent loan balance		amount towards principal		new loan balanc

8. How much of the second payment is interest and how much pays down the loan?

owed before

Interest owed is $ 764.51 * 0.04 = 30.58.

pays it down

Amount towards principal is $ 275.49 − 30.58 = 244.91.

9. What is the outstanding balance on the loan after the second payment? $ $519.60

10. An **amortization table**

End Period	Payments Remaining	Payment Amount	Amt To Interest	Amt To Principal	Outstanding Principal
0	4	n/a	n/a	n/a	$1000
1	3	$ 275.49	$ 40	$ 235.49	$ 764.51
2	2	$ 275.49	$ 30.58	$ 244.91	$ 519.60
3	1	$ 275.49	$ 20.78	$ 254.71	$ 264.89
4	0	$ 275.49	$ 10.60	$ 264.89	$ 0

To find the outstanding balance at any time during the loan the TVM Solver can be used. Change the number of payments, N, to the number of remaining payments and solve for the new PV.

11. What is your outstanding loan balance after 9 months?

a. How many payments are remaining after 9 months? 1

b. What is the outstanding principal? 264.89

When a loan is used to purchase a significant item (like a car or house), as you pay off the loan, you gain equity in the item. The **equity** is the dollar amount of the item that belongs to you. The rest of the value of the item belongs to the bank (the amount you still owe). That is, if an item is sold at a particular time, your equity is the money you would have after paying off the remaining amount on your bank loan:

Part III

A house is typically a person's largest investment. The loan on a house is called a mortgage. The equity in your home is the value of the home at the present time minus the amount owed to the bank on the mortgage.

Suppose you want to buy a house that costs $175,000.

$17,500

Finance: Annuities and Amortization

1. If you make a 10% down payment, how much do you need to finance? $157,500 = 175,000 − 17,500

N = 30×12, I = 6, PV = ↓, PMT = ?, FV = 0, P/Y = 12

2. If you finance with a 30-year mortgage at 6% annual interest compounded monthly on the unpaid balance, how large are the monthly payments and how much interest is paid in all?

Payments are $ 944.29

Total interest is $ (944.29)(360) − 157,500 = $182,444.40

3. If you finance with a 20-year mortgage at 6% annual interest compounded monthly on the unpaid balance, how large are the monthly payments and how much interest is paid in all?

Payments are $ 1128.38

Total interest is $ 113,311.20

Note: In **2.**, you will have a lower monthly payment, but pay more in interest in all. In **3.**, you will have a higher monthly payment, but pay less interest in all.

4. You choose the 30-year mortgage at 6% annual interest compounded monthly for 30 years. Your equity when you move in is your down payment:

175 000	−	157,500	=	17,500
value of house (stays the same)		amount owed to bank		equity

$944.29

Finance: Annuities and Amortization

5. How much of your first payment is interest and what is your outstanding loan balance after your first payment?

157,500 (amount owed) × $\left(\frac{.06}{12} = 0.005\right)$ (monthly interest rate) = 787.50 (monthly interest owed)

944.29 (payment amount) − 787.50 (monthly interest owed) = 156.79 (amount to princip)

157,500 (current loan balance) − 156.79 (amt to principal) = 157,343.51 (new loan balance)

How much is the interest owed on the 2nd payment?

6. This type of information can be organized in an amortization table: INT = $157{,}343.51 * \left(\frac{.06}{12}\right) = 786.72$

End of Period	Payments Remaining	PMT Amt	Amt To Interest	Amt To Princ	Outst. Princ	Equity
0	360	n/a	n/a	n/a	$ 157,500	$ 17,500
1	$	$ 944.29	$ 787.50	$ 156.79	$ 157,343.51	$ 17,656.79
2	$	$ 944.29	$ 786.72	$ 157.57	$ 157,185.64	$ 17,814.36

7. After 10 years you want to refinance the house. How much do you owe the bank at this point? In other words, what is your outstanding principal? $131,804.73

N = # of PMTS REMAINING = 360 - 12x10 = 240 PMTS left

N=240 PMT= 944.29
I=6 FV= 0
PV=? P/Y=12

8. What is your equity after 10 years? $43,195.27
(Assume the value of the house is still $175,000)
- 131,804.73