## Annuities and Amortization

The most common compounding frequencies are:

| Compounding | Annually | Semi-Annually | Quarterly | Monthly | Weekly | Daily |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| \# of times per year (m) | 1 | 2 | 4 | 12 | 52 | 365 |

## Part I

1. Ben would like to have $\$ 60,000$ available for college in 18 years.
a. If an account exists earning 7\% annual interest compounded annually, how much should be deposited now to have the $\$ 60,000$ in 18 years?
\$ $\qquad$
b. How much interest is earned?
\$ $\qquad$

If you don't have all the money available now, you can save a bit each year to reach this goal. This is a type of annuity.

An annuity is an account into which regular payments are made. The annuities in this class are all CERTAIN and SIMPLE, meaning:

- The payments are made at fixed time intervals.
- The periodic payments are of equal size.
- The payments are made at the end of the interval.
- The interest is paid at the end of the interval.
c. Use the TVM Solver to find the amount of the annual payments needed to reach the goal of $\$ 60,000$.
$\mathrm{N}=$ $\qquad$ $\mathrm{I} \%=$ $\mathrm{PV}=0 \quad \mathrm{PMT}=$ $\qquad$ $\mathrm{FV}=$ $\qquad$ $\mathrm{P} / \mathrm{Y}=\mathrm{C} / \mathrm{Y}=$ $\qquad$

The amount of the annual payments is $\qquad$ .
d. In total, how much money was deposited into the account? $18 * \mathrm{PMT}=$ $\qquad$
e.
$\overline{\text { interest earned }}^{=} \quad \overline{\text { final value of account }}$ amount depositied
f. Why are the interest amounts in parts $\mathbf{b}$. and $\mathbf{e}$. different?
2. You are saving for retirement by depositing $\$ 400$ per month in an account that earns $8 \%$ annual interest compounded monthly. Determine the total amount in the account and the total interest earned if this is done for 10 years, 30 years and 50 years.

After 10 years, \$ $\qquad$ total and \$ $\qquad$ in interest

After 30 years, \$ $\qquad$ total and \$ $\qquad$ in interest

After 50 years, \$ $\qquad$ total and \$ $\qquad$ in interest

Loans also deal with making payments. A bank loans you money and you make regular payments until you no longer have any of the bank's money. This means that the original value of the loan is the present value $(\mathrm{PV})$ and the future value of the loan ( FV ) is 0 .
3. You find a car advertised for no money down with payments of $\$ 199$ per month for 72 months. In the fine print you see that the interest is $9 \%$ annual interest compounded monthly on the unpaid balance.
a. What is the purchase price of the car? That is, how large was the loan you took out? \$ $\qquad$
b. How much money total will you pay out of pocket? $72 * 199=$ $\qquad$
c. Interest Paid = Amount of Money Repaid - Amount Borrowed = $\qquad$
4. Your goal is to retire when you become a multi-millionaire ( $\$ 2,000,000$ ) so you start saving $\$ 100$ per week in an account that pays $10 \%$ annual interest compounded weekly. If you start at age 23 , how old will you be when you retire?
5. A cello costs $\$ 998$. You pay off the cello by making payments of $\$ 39.00$ per month for 3 years.
a. What annual interest rate compounded monthly were you charged (rounded to 4 decimal places)?
b. How much interest do you pay in total? \$ $\qquad$

## Part II

You take out a $\$ 1000$ loan. The terms of the 1-year loan are $16 \%$ annual interest compounded quarterly on the unpaid balance.

1. How much are the quarterly payments? $\$$ $\qquad$
2. How much interest is paid in total? \$ $\qquad$
3. If the interest was compounded annually, instead, and you just paid the entire loan off at the end of the year, how much would the total interest be?
\$ $\qquad$
4. Why are the answers in 2 . and 3 . different?

Each payment is broken down into two parts; the interest owed is paid first and then the rest of the payment goes towards paying off (paying down) the amount owed (principal).


The amount of interest paid is the largest in your first payment, while nearly all of the last payment goes towards the principal. Even though the payment is the same each month, the distribution of interest and principal changes with every payment.
5. What is the periodic, in this case quarterly, interest rate? $\qquad$ \% Hint: There are 4 quarters in a year.

