Vector Functions (Section 1.3)

A curve of the type \( x = x(t), \ y = y(t) \) is called a parametric curve and the variable \( t \) is the parameter.

**EXAMPLE 1**
Graph the parametric function \( x = t^3 - 2t, \ y = t^2 - t \)

\[ \begin{array}{c}
\text{(a)} \ x = 2t - 1, \quad y = 2 - t, \quad -3 \leq t \leq 3 \\
\text{(b)} \ x = 2t - 1, \quad y = t^2 - 1
\end{array} \]

**EXAMPLE 2**
Sketch the curve represented by the parametric equations and then eliminate the parameter to find the Cartesian equation of the curve.

\[ \begin{array}{c}
\text{(a)} \ x = 2t - 1, \quad y = 2 - t, \quad -3 \leq t \leq 3 \\
\text{(b)} \ x = 2t - 1, \quad y = t^2 - 1
\end{array} \]
For each value of the parameter $t$ we may view the point $(x(t), y(t))$ on a parametric curve as the endpoint of a vector $\mathbf{r}(t) = \langle x(t), y(t) \rangle = x(t)\mathbf{i} + y(t)\mathbf{j}$

**EXAMPLE 3**
Describe the motion of a particle with position $(x, y)$ or $\mathbf{r}(t)$ as $t$ varies in the given interval.
(a) $\mathbf{r}(t) = (8t - 3)\mathbf{i} + (2t - 1)\mathbf{j}$, $0 \leq t \leq 1$

(b) $\mathbf{r}(t) = \langle 2\sin t, 3\cos t \rangle$, $0 \leq t \leq 2\pi$

Consider a line $L$ as shown. Can we write this as a vector $\mathbf{r}(t)$?

The vector equation of a line is given by $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$ where $\mathbf{r}_0$ is a position vector to a point on the line, $\mathbf{v}$ is a vector parallel to the line, and $t$ is a scalar.

**EXAMPLE 4**
Given the points $(3, 4)$ and $(2, 8)$, find a vector equation and a parametric equations for the line that passes through these two points.
EXAMPLE 5
Given the point \( P(2, 5) \) and vector \( \mathbf{a} = \langle 3, 0 \rangle \), find
(a) a vector equation
(b) parametric equations
(c) a Cartesian equation for a line that passes through the point \( P \) and is parallel to \( \mathbf{a} \).

EXAMPLE 6
Determine if the lines below are parallel, perpendicular or neither. If the lines are not parallel, find the point of intersection
\( L_1 : \mathbf{r}(t) = \langle -4 + 2t, 5 + t \rangle \)
\( L_2 : \mathbf{r}(t) = \langle 2 + 3t, 4 - 6t \rangle \)

EXAMPLE 7
An object is moving in the \( xy \)-plane and its position after \( t \) seconds is \( \mathbf{r}(t) = \langle t - 3, t^2 - 2t \rangle \)
(a) Find the position of the object at time \( t = 5 \).
(b) At what time does the object pass through the point \((1, 8)\)?
(c) Does the object pass through the point \((3, 20)\)?
(d) Find an equation in \( x \) and \( y \) whose graph is the path of the object.