Absolute Extreme values of $f$
- A function $f$ has an absolute (or global) maximum at $c$ if $f(c) \geq f(x)$ for all $x$ in the domain of $D$. The number $f(c)$ is called the maximum value of $f$ on $D$.
- A function $f$ has an absolute (or global) minimum at $d$ if $f(d) \leq f(x)$ for all $x$ in the domain of $D$. The number $f(d)$ is called the minimum value of $f$ on $D$.

Local Extreme values of $f$
- A function $f$ has a local (or relative) maximum at $c$ if $f(c) \geq f(x)$ when $x$ is near $c$.
- A function $f$ has a local (or relative) minimum at $d$ if $f(d) \leq f(x)$ when $x$ is near $c$.

The Extreme Value Theorem
If $f$ is continuous on a closed interval $[a,b]$, the $f$ attains an absolute maximum value of $f(c)$ and an absolute minimum value $f(d)$ at some numbers $c$ and $d$ in $[a,b]$.

EXAMPLE 1
The graph of the function $f$ with domain $[-5,5]$ is shown.
(a) What are the absolute extrema?
(b) What are the local extrema?

EXAMPLE 2
Find all absolute and local extrema for the following functions

$$f(x) = 1 - x^2, \quad 0 \leq x \leq 1$$

$$f(x) = 1 - x^2, \quad -1 \leq x \leq \frac{1}{2}$$
EXAMPLE 3
Find all critical numbers for the following functions

\[ f(x) = x^3 + 6x^2 + 3x - 1 \]

\[ f(x) = |x^2 - 1| \]

\[ f(x) = \frac{x + 1}{x^2 + x + 1} \]

\[ f(x) = x + \sin x \]

\[ f(x) = xe^{2x} \]

\[ f(x) = \sqrt{x^2 - x} \]

**Fermat’s Theorem**
If \( f \) has a local maximum or minimum at \( c \), and if \( f'(c) \) exists, then \( f'(c) = 0 \).

**Critical Numbers**
A critical number (or value) of a function \( f \) is a number \( c \) in the domain of \( f \) such that either \( f'(c) = 0 \) or \( f'(c) \) does not exist.
Finding Absolute Extrema
To find the absolute extrema of a continuous function \( f \) on a closed interval \([a, b]\),

1. Find all critical values \( c_1, c_2, \ldots \) of \( f \) in the interval \((a, b)\)
2. Find the values of \( f(c_1), f(c_2), \ldots \) along with \( f(a) \) and \( f(b) \)
3. The absolute maxima is the largest of the values found in steps 1 and 2.
4. The absolute minima is the smallest of the values found in steps 1 and 2.

EXAMPLE 4
Find the absolute extrema for the following functions

i. \( f(x) = x^3 - 12x + 1, \quad [-3, 5] \)

ii. \( f(x) = -x^3 + 27x + 1, \quad [0, 4] \)

iii. \( f(x) = x - 2 \cos x, \quad [0, \pi] \)

iv. \( f(x) = x - 2 \cos x, \quad [-\pi, \pi] \)

v. \( f(x) = \frac{x}{x+1}, \quad [1, 2] \)

vi. \( f(x) = \frac{x}{x+1}, \quad [-2, 2] \)

vii. \( f(x) = \frac{\ln x}{x}, \quad [1, 3] \)