Antiderivatives (Section 5.7)

The derivative of what function \( F(x) \) will give the function \( f(x) = 2x \)? Since the derivative of \( x^2 \) is \( 2x \), we can say the function \( F(x) = x^2 \) is the antiderivative of \( 2x \).

**EXAMPLE 1**

Find the most general antiderivative for the following functions

(i) \( f(x) = x^3 - 4x^2 + 17 \)

(ii) \( f(x) = \sqrt[3]{x^2} - \sqrt{x^3} \)

(iii) \( f(x) = \frac{x + x^2 - 1}{x^3} \)

(iv) \( f(x) = e^x + \frac{4}{\sqrt{1-x^2}} \)

**TABLE OF ANTIDERIVATIVES**

<table>
<thead>
<tr>
<th>Function</th>
<th>Antiderivative</th>
<th>Function</th>
<th>Antiderivative</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>( kx + c )</td>
<td>( \sec x \tan x )</td>
<td>( \sec x + c )</td>
</tr>
<tr>
<td>( x^n, n \neq -1 )</td>
<td>( \frac{x^{n+1}}{n+1} + c )</td>
<td>( \sec^2 x )</td>
<td>( \tan x + c )</td>
</tr>
<tr>
<td>( x^{-1} )</td>
<td>( \ln</td>
<td>x</td>
<td>+ c )</td>
</tr>
<tr>
<td>( e^x )</td>
<td>( e^x + c )</td>
<td>( -\csc x \cot x )</td>
<td>( -\csc x + c )</td>
</tr>
<tr>
<td>( \cos x )</td>
<td>( \sin x + c )</td>
<td>( \frac{1}{x^2 + 1} )</td>
<td>( \arctan x + c )</td>
</tr>
<tr>
<td>( \sin x )</td>
<td>( -\cos x + c )</td>
<td>( \frac{1}{\sqrt{1-x^2}} )</td>
<td>( \arcsin x + c )</td>
</tr>
</tbody>
</table>
EXAMPLE 2
Find \( f(x) \) given that

(i) \( f'(x) = 12x^2 - 24x + 1 \) and \( f(1) = -2 \)

(ii) \( f''(x) = 3e^x + 4\sin x \), \( f(0) = 1 \), and \( f'(0) = 2 \)

EXAMPLE 3
A particle is moving with acceleration \( a(t) = 3t + 8 \text{ m/s}^2 \). Find the position \( s(t) \) of the object at time \( t \) if we know \( s(0) = 1 \) and \( v(0) = -2 \)

EXAMPLE 4
A stone is thrown downward from a 450m tall building at a speed of 5 m/s. Find a formula for the distance of the stone above ground.
**EXAMPLE 5**

A car braked with constant deceleration of 40 ft/s². The skid marks produced were 160 ft before the car came to a stop. How fast was the car traveling when the brakes were first applied?

A vector function \( \mathbf{R}(t) = X(t)\mathbf{i} + Y(t)\mathbf{j} = \langle X(t), Y(t) \rangle \) is called the antiderivative of \( \mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} = \langle x(t), y(t) \rangle \) on an interval \( I \) if \( \mathbf{R}'(t) = \mathbf{r}(t) \) for all \( t \) in \( I \).

That is, \( X'(t) = x(t) \) and \( Y'(t) = y(t) \).

The most general antiderivative of \( \mathbf{r} \) on \( I \) is \( \mathbf{R}(t) + \mathbf{C} \) where \( \mathbf{C} \) is an arbitrary constant vector.

**EXAMPLE 6**

A projectile is fired from a position 200m above the ground with an initial speed of 500m/s and an angle of elevation of 30° above the horizontal. Find a vector equation for the position of the projectile at time \( t \) in seconds.