Finance

1. What is the amount repaid on a loan of $600, if the loan charges 8.6% compounded continuously for 44 months? How much would be repaid if the loan charged simple interest?

2. A credit card charges 24% annual interest compounded monthly on the unpaid balance. You owe $4000 on this credit card. To pay it off, you stop using it and make monthly payments of $100.
   (a) How long until the card is paid off?
   (b) How much was paid in interest charges?
   (c) How much of the first payment was interest?
   (d) How much of the second payment was interest?
   (e) How much is still owed on the credit card after two years of making payments?

3. You find that you can afford a monthly mortgage payment of $1600. You find a loan that charges 6% annual interest compounded monthly on the outstanding balance for 25 years. How large of a loan can you get?

4. A lottery prize is $20,000,000 which is paid in 20 annual payments of $1,000,000 each. If the winner is given $1,000,000 now, how much needs to be deposited in an account paying 8.5% annual interest compounded annually if the winner is to be paid $1,000,000 per year at the end of the year for the next 19 years?

5. To save for retirement you make semi-annual payments of $3000 to an account that pays 9% annual interest compounded semi-annually. How much will you have after saving for 10 years? For 20 years? How much interest is earned in the first half of the 11th and 21st years?

6. The Bank of the North offers a savings account that pays 2.6% interest, compounded quarterly. The Bank of the East offers a similar savings account that pays 2.58%, compounded daily. Compute the effective rate for each bank, and determine which bank offers the better deal for an investor.

7. Joey purchases 100 shares of a particular stock for a total of $20,000 in 2006. He sold the shares in 2008 for $22,222. Determine the effective annual rate of return on the investment, rounded to two decimal places.

8. You purchase $220,000 house by making a down payment of 20% of the purchase price and finance the remainder for 40 years at 6.3% annual interest that is compounded monthly on the unpaid balance for 40 years.
   (a) How much are the monthly payments and how much interest is paid in all?
   (b) How much of the first payment is interest and how much is used to pay off the loan?
   (c) What is your equity after 10 years?
   (d) How much are the monthly payments and how much interest is paid in all the loan is for 30 years?
   (e) How much are the monthly payments and how much interest is paid in all the loan is for 20 years?

9. You deposited $50 fifty one years ago in an account that pays 4% annual interest compounded daily. How much is in the account now?

Systems of Linear Equations

1. What is the solution to the augmented matrix

\[
\begin{bmatrix}
1 & 0 & -2 & 0 & 0 \\
0 & 1 & 1 & 1 & -3 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

2. Which of the following are particular solutions to the parametric solution \((3t - 7, -2s + l, s, t)\)?
   A. \((-7,1,0,0)\)   B. \((-4,1,0,1)\)   C. \((-7,1,1,0)\)   D. \((-4,1,1,1)\)   E. None of these

3. A company is buying three kinds of vehicles. Carts hold 3 people and cost $9,000, vans hold 8 people can cost $27,000 and minivans hold 7 people and cost $27,000. The company needs to seat 48 people and has $162,000 to purchase vehicles. How many of each type of vehicle can be purchased?
4. A cook has 15 pounds of nuts to make brownies and fudge. Each batch of brownies uses 1 pound of nuts and makes 32 brownies. Each batch of fudge uses 1.5 pounds of nuts and makes 24 pieces of fudge. The cook also wants twice as many pieces of fudge as brownies. How much of each item should be made?

\[
\begin{align*}
6x + z &= 5 \\
6x + 9y &= 5
\end{align*}
\]

5. Solve the systems

\[
\begin{align*}
9x + 1.5y &= 8 \\
3x + 18y &= 8.5
\end{align*}
\quad \text{and} \quad \begin{align*}
9x + 1z &= 8 \\
3x + 4.5y &= 9
\end{align*}
\]

6. A merchant wishes to mix three kinds of coffee, Columbian (selling for $8 per pound), Kona ($10 per pound) and Blue Mountain ($15 per pound) to get 50 pounds of a mixture that can be sold for $11.70 per pound. The amount of the Columbian must be 3 pounds more than the amount of Kona. Find the number of pounds of each that will be used.

**Matrices**

1. Let matrix \( A \) represent the number of bicycles in stock of brands Tiger (T) and Snake (S) at stores in Paddington (P) and Quentin (Q). Let matrix \( B \) represent the proportion of bikes from brands T and S that are road bikes (R) and mountain bikes (M).

\[
\begin{bmatrix}
P & Q \\
T & S
\end{bmatrix} = \begin{bmatrix}
60 & 75 \\
45 & 50
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
R & M \\
T & S
\end{bmatrix} = \begin{bmatrix}
0.5 & 0.8 \\
0.5 & 0.2
\end{bmatrix}
\]

Consider the product of these two matrices, called \( C \). Which of the following is a true statement about \( C \)?

A. \( C = AB \) and represents the number of road and mountain bikes in stock at each store.
B. \( C = BA \) and represents the number of road and mountain bikes in stock at each store.
C. \( C = AB \) and represents the number of Tiger and Snake brand bikes in stock at each store.
D. \( C = BA \) and represents the number of Tiger and Snake brand bikes in stock at each store.
E. None of these

2. Solve the following matrix equation for \( X \): 

\[
2X + D = XB.
\]

3. Determine the values of \( c \) and \( d \) in the following matrix equation:

\[
\begin{bmatrix}
a & -2 \\
2 & d
\end{bmatrix}^T + 4 \begin{bmatrix}
a \\
c
\end{bmatrix} = \begin{bmatrix}
2 & 4 \\
-6 & -3
\end{bmatrix}
\]

4. Matrix \( F \) shows the number of packages of fresh herbs sold at three different stores. The basil sells for $2.00 per package, the rosemary for $3.00 per package and the sage for $2.50 per package. Find a matrix \( G \) such that when it is multiplied by \( F \) will give the total amount of revenue from selling herbs at each store.

\[
F = \begin{bmatrix}
\text{basil} & \text{rosemary} & \text{sage} \\
\text{Store A} & 11 & 3 & 9 \\
\text{Store B} & 5 & 9 & 11 \\
\text{Store C} & 14 & 2 & 17
\end{bmatrix}
\]

5. The matrices \( A, B, C, D \) and \( I \) have the following properties:

<table>
<thead>
<tr>
<th>Matrix</th>
<th>Size</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>6x7</td>
<td>6x6 singular</td>
</tr>
<tr>
<td>( B )</td>
<td>7x6</td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>6x6x6</td>
<td></td>
</tr>
<tr>
<td>( D )</td>
<td>7x7x7</td>
<td></td>
</tr>
<tr>
<td>( I )</td>
<td>6x6</td>
<td>6x6 identity</td>
</tr>
</tbody>
</table>

Determine the dimensions of the following matrices. If the matrix does not exist, write DNE for the dimensions.

a. \( A + B \) is a \[ \text{[ ]} \] matrix
b. \( C + I \) is a \[ \text{[ ]} \] matrix
c. \( AC^{-1} \) is a \[ \text{[ ]} \] matrix
d. \( D^{-1}B \) is a \[ \text{[ ]} \] matrix
e. \( IA \) is a \[ \text{[ ]} \] matrix
f. \( BC \) is a \[ \text{[ ]} \] matrix

6. The economy of the stone-age village Bedrock has three industries, stone cutting (S), farming (F), and hunting (H). The input-output matrix is given below and the demand from the local city of Rock Vegas is $1500 of stone, $6500 of farming and $4000 of hunting. How much of stone, farming, and hunting needs to be produced in total to meet all demands? What is the meaning of the entry \( a_{12} \)?

\[
A = \begin{bmatrix}
S & F & H \\
0.3 & 0.3 & 0.25 \\
0.2 & 0.25 & 0.3 \\
0.2 & 0.1 & 0.2
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
S & F & H \\
0.3 & 0.3 & 0.25 \\
0.2 & 0.25 & 0.3 \\
0.2 & 0.1 & 0.2
\end{bmatrix}
\]

\[ A = \begin{bmatrix}
S & F & H \\
0.3 & 0.3 & 0.25 \\
0.2 & 0.25 & 0.3 \\
0.2 & 0.1 & 0.2
\end{bmatrix} \]

\[ H = \begin{bmatrix}
S & F & H \\
0.3 & 0.3 & 0.25 \\
0.2 & 0.25 & 0.3 \\
0.2 & 0.1 & 0.2
\end{bmatrix} \]