1.6 Conditional Probability

A survey is done of people making purchases at a gas station. Most people buy gas (Event $A$) or a drink (Event $B$).

<table>
<thead>
<tr>
<th>buy drink ($B$)</th>
<th>no drink ($B^c$)</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>buy gas ($A$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>no gas ($A^c$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>total</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What is the probability that a person bought gas and a drink?

What the probability that a person who buys a drink also buys gas? In other words, given that a person bought a drink ($B$), what is the probability that they bought gas ($A$)?

Notation: $P(E \mid F) =$ the probability of $A$ given $B$

The conditional probability of event $E$ given event $F$ is

What is the probability that a person who buys gas also buys a drink?
The Product Rule:

Example
At a party, 1/3 of the guests are women. 75% of the women wore sandals and 20% of the men wore sandals.

a) What is the probability that a person chosen at random at the party is a man wearing sandals?

b) What is the probability that a randomly chosen guest is wearing sandals?

Example
Consider drawing 3 cards from a standard deck of 52 cards without replacement.

a) What is the probability that the three cards are hearts?

b) What is the probability that the third card drawn is a heart given the first two cards are hearts?
**Example**
A bag has 3 silver and 4 copper coins. A pouch has 1 silver and 2 copper coins. A coin is drawn at random from the bag and placed in the pouch. A coin is then drawn from the pouch. What is the probability that a silver coin is drawn from the pouch given that a silver coin was chosen from the bag?

**Example**
A medical test has been developed to detect $xyzzy$ disease. It is estimated that 5% of the patients who come in for the test have the disease. When the test is given to a patient who has $xyzzy$ disease, it is detected (positive) 90% of the time. When given to a patient who does not have $xyzzy$ disease, a positive result is returned 15% of the time. What is the probability that a person has $xyzzy$ disease and tests negative?
**Independent Events:** Events $E$ and $F$ are independent if $P(E \mid F) = P(E)$

**Example**
A medical experiment showed the probability that a new medicine was effective was 0.75, the probability of a certain side effect was 0.4 and the probability for both occurring is 0.3. Are these events independent?

**Example**
The side effects of a certain medicine include a 25% chance of headaches and 30% chance of fatigue. What is the probability that a person taking this medicine will suffer exactly one of these side effects if they are independent of each other?
1.7 Bayes’ Theorem

Given $P(E | F)$, can we find $P(F | E)$?

**Example**
We are to choose a marble from a cup or a bowl. We need to flip a coin to decide to choose from the cup or the bowl. The bowl contains 1 red and 2 green marbles. The cup contains 3 red and 2 green marbles. What is the probability that a marble came from the bowl given that it is red?

**Example**
A survey of the local middle school found the percent of students in each grade who own a calculator. The results are below. What is the probability that a student with a calculator is in the 5th grade?

<table>
<thead>
<tr>
<th>Grade</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent of student body</td>
<td>37</td>
<td>32</td>
<td>31</td>
</tr>
<tr>
<td>Percent that own a calculator</td>
<td>13</td>
<td>28</td>
<td>59</td>
</tr>
</tbody>
</table>
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