Chapter F Finance

Section F.1 Simple Interest and Discount

Simple interest – interest computed as a percentage of the principal

Simple interest earned: \( I = Prt \)

\( I = \) interest earned
\( P = \) principal
\( r = \) interest rate (decimal form)
\( t = \) time period (in years)

Future value: \( F = P + Prt = P(1 + rt) \)

Example
If a bank loans $525 to an individual for 3\( \frac{1}{2} \) years at 7.25% simple interest, what will be the amount repaid on the loan?

\[
F = 525 \left( 1 + 0.0725 \times 3.5 \right) = 658.21875
\]

\[
\text{\$658.22}
\]

Example
If a $1000 deposit grows in value to $1024 after 9 months, what is the simple interest rate that is earned?

\[
I = 1024 - 1000 = 24 = 1000 \times r \times \frac{9}{12} = 750r
\]

\[
r = \frac{24}{750} \approx 0.032 \quad \text{or} \quad 3.2\%
\]
**Example**

How much should be placed in an account that pays simple interest of 4\% so that the value of the account after 18 months is $3000? 

\[
F = P \left(1 + rt\right) \\
3000 = P \left(1 + .04 \times 1.5\right) = 1.06 \cdot P \\
P = \frac{3000}{1.06} = 2830.19 \rightarrow \pound 2830.19
\]

**Example**

You have an account with $500 that pays 5\% simple interest. How long until your account doubles in value?

\[
F = P \left(1 + rt\right) \\
1000 = \frac{500}{500} \left(1 + .05 \cdot t\right) \\
\frac{1000}{500} = 1 + .05 \cdot t \\
2 = 1 + .05 \cdot t \\
1 = .05 \cdot t \\
t = \frac{1}{.05} = 20 \text{ years}
\]

When a loan is discounted, the interest owed is deducted when the loan is made. The interest deducted is called the **discount** (D), the amount the borrower actually receives is called the **proceeds** (P) and the amount to be repaid is called the **maturity value** (M). So 

\[D = Mrt\text{ and } P = M - D\]

Where \( t \) is in years and \( r \) is the annual simple interest rate.

**Example**

A borrower gets a loan in which she agrees to pay the bank $4000 in 8 months at 6\% simple discount. What is the discount and what are the proceeds?

\[
D = 4000 \times .06 \times \frac{8}{12} = \pound 160 = D \\
P = 4000 - 160 = \pound 3840 = P
\]
**Example**

If a borrower wants to have proceeds of $4000 on a discounted loan for 8 months at 6%, what is the maturity value of the loan?

\[
P = 4000 = M - D = M - M \cdot \left(0.06 \cdot \frac{8}{12}\right) = 4000
\]

\[
4000 = M \left(1 - 0.06 \cdot \frac{8}{12}\right) = 0.96M
\]

\[
M = \frac{4000}{0.96} = \frac{4166.67}{0.96} = M
\]

**Example**

A borrower gets a loan in which she agrees to pay the bank $4000 in 8 months at 6% simple discount. What is the effective interest rate?

\[
I = 160 = 3840 \times r_{\text{eff}} \times \frac{8}{12}
\]

\[
r_{\text{eff}} = \frac{160}{3840 \times \frac{8}{12}} = 0.0625 \text{ or } 6.25\%
\]

The effective rate of interest on a discounted loan of length \( t \) years with a discount rate of \( r \) is \( r_{\text{eff}} = \frac{r}{1 - rt} \)
Section F.2 Compound Interest

When interest is paid periodically and the interest earns interest, we have compound interest.

**Example**
You deposit $100 into an account that pays 10% annual interest that is compounded annually. How much is in the account after 4 years?

\[
F = P \left(1 + \frac{r}{m}\right)^{mt} = 100 \left(1 + \frac{0.10}{1}\right)^{1 \times 4} = 144.0
\]

\[
F_1 = 100 \left(1 + \frac{0.10}{1}\right) = 100 \times 1.1 = 110
\]

\[
F_2 = 110 \left(1 + \frac{0.10}{1}\right) = 110 \times 1.1 = 121
\]

\[
F_3 = 121 \left(1 + \frac{0.10}{1}\right) = 121 \times 1.1 = 133.1
\]

\[
F_4 = 133.1 \left(1 + \frac{0.10}{1}\right) = 133.1 \times 1.1 = 146.41
\]

where \( m \) is the number of compounding periods per year and \( r \) is the annual interest rate as a decimal.

**Example**
You deposit $300 into an account that pays 6% annual interest compounded monthly. How much is in the account after 3 months?

\[
F_1 = 300 \left(1 + \frac{0.06}{12}\right) = 300 \left(1.005\right) = 301.05
\]

\[
F_2 = 301.05 \left(1.005\right) = 301.05 \times 1.005 = 303.0075
\]

\[
F_3 = 303.0075 \left(1.005\right)^2 = 306.0552
\]