1. (3 points) A student has a quiz in his math class. He can study hard for the quiz, study a bit for the quiz or not study at all. The quiz can be challenging, moderate or easy. The student assigns a “satisfaction value” to each of the outcomes as shown in the matrix below:

<table>
<thead>
<tr>
<th>study hard</th>
<th>study a little</th>
<th>don't study</th>
</tr>
</thead>
<tbody>
<tr>
<td>challenging</td>
<td>moderate</td>
<td>easy</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

a) What should the student do if he is an optimist?

*Study hard (and expect a challenging quiz)*

b) What should the student do if he is a pessimist?

*Study hard (and expect a challenging quiz)*

c) What is the expected value of studying a little if there is a 25% chance of a challenging quiz, 50% chance of a moderate quiz and 25% chance of an easy quiz?

\[
E = 2(0.25) + 4(0.5) + 1(0.25) = 2.75
\]

2. (1 point) Remove any dominated strategies from the matrix

\[
\begin{pmatrix}
5 & 4 & 1 \\
-4 & 0 & 3
\end{pmatrix}
\]

3. (1 point) Bill and Sue play a game with coins. Both flip a coin at the same time. If both coins show heads or both coins show tails, Bill wins $2 from Sue. If one coin shows heads and one shows tails, Sue wins $2 from Bill. Construct the payoff matrix for this game.

\[
\begin{pmatrix}
-2 & -2 \\
-2 & 2
\end{pmatrix}
\]

or

\[
\begin{pmatrix}
4 & -2 \\
-2 & -2
\end{pmatrix}
\]

\[
\begin{pmatrix}
-2 & 2 \\
2 & -2
\end{pmatrix}
\]
4. (2 points) Using the payoff matrix below for a two-person zero-sum game, find the saddle point, the optimal strategy for the row player and the value of the game. Is the game fair? If not, why not?

\[
\begin{pmatrix}
1 & -4 & 5 \\
-2 & 0 & -1 \\
4 & 1 & 1 \\
-1 & -2 & -4
\end{pmatrix}
\]

Saddle @ row 3 and col 2.
Row player play row 3 all the time.
Value of the game is 1.
Not fair as it favors 1.

5. (3 points) Given the payoff matrix \( M = \begin{pmatrix} 2 & -3 \\ -5 & 6 \end{pmatrix} \).

(a) What is the value of this game if the row player plays row 1 20% of the time and the column player plays column 1 70% of the time?

(b) What is the row player’s optimum strategy?

(c) What is the value of the game if each player plays their optimum strategy?

(a) \((.2 \rightarrow .8) \begin{pmatrix} 2 & -3 \\ -5 & 6 \end{pmatrix} \begin{pmatrix} 7 \\ 3 \end{pmatrix} = \begin{pmatrix} -1.26 \end{pmatrix}\)

Value is \(-1.26\).

(b) \(q_1 = \frac{6 - (-5)}{2 + 6 - (-3) - (-5)} = \frac{11}{16} = .6875\)

\(q_2 = 1 - q_1 = .3125\)

Play row 1 .6875 of the time and row 2 .3125 of the time.

(c) \(q_1 = \frac{6 - (-3)}{2 + 6 - (-3) - (-5)} = \frac{9}{16} = .5625\)

\(q_2 = 1 - q_1 = .4375\)

\(E = (.6875, .3125) \begin{pmatrix} 2 & -3 \\ -5 & 6 \end{pmatrix} \begin{pmatrix} .5625 \\ .4375 \end{pmatrix} \times (-, .1875)\)

\(p_1 = \frac{a_{12} - a_{22}}{a_{11} + a_{12} - a_{21} \neq a_{22}}\)

\(q_1 = \frac{a_{21} - a_{11}}{a_{11} + a_{21} - a_{12} \neq a_{11}}\)