LOGIC

1. Which of the following are statements?
   - Math is hard.
   - There was measurable rainfall yesterday at the airport.
   - How hot is it?

2. Translate the sentence into symbolic form: "The food is sweet and it is not spicy".

3. Write a truth table for \((p \lor q) \land \neg(q \land r)\)

4. Write the negation for each of the statements below:
   (a) The car is blue
   (b) No cars are blue
   (c) Some cars are blue

5. Write the compound statement \(\neg (p \lor q)\) as a sentence given that
   \(p: \) The birds are hungry
   \(q: \) The birds are thirsty

- Not a statement – need to define "hard"
  A statement
- Not a statement – a question
- \(p: \) the food is sweet
  \(q: \) the food is spicy
  \(p \land \neg q\)
- \(\begin{array}{c|ccc|c}
  p & q & r & p \lor q & (p \lor q) \land (q \land r) \\
  \hline
  T & F & T & F & F \\
  T & F & F & F & F \\
  F & T & T & T & T \\
  F & T & F & T & T \\
  F & F & T & F & F \\
  F & F & F & F & F \\
\end{array}\)
- a) The car is not blue
  b) At least one car is blue
  c) No cars are blue
  d) \(\mp\) below
- Either the birds are not hungry or they are thirsty
  \(\text{NNN, NBB, NNB} \iff \text{BBB}\)
- Some cars are not blue \(\iff\) All cars are blue
SETS

1. Express the shaded region in set notation:

2. A class of math students can be grouped into the following sets:
   \( A = \{ x | x \text{ is a woman} \} \)
   \( B = \{ x | x \text{ has taken Economics} \} \)

   Find the set of women who have not taken Economics in set builder notation.

3. A store has sold 100 microwaves. 80 of the microwaves have turntables and 40 of them have programs. If 90 of them have programs or turntables, how many have only turntables?

4. A survey of two hundred students is done at a school cafeteria. Use the information given to fill in a Venn diagram:
   - 55 students like pizza and burritos.
   - 130 students did not like chicken.
   - 30 students like all three items.
   - 35 students like burritos but did not like chicken.
   - 55 students like only pizza.
   - 60 students like exactly 2 of these dishes.
   - 15 students like chicken and pizza but not burritos.

5. Shade the region corresponding to
   (a) \( \{ x | x \notin A \text{ or } x \in B \} \)
   (b) \( (A \cup B)^c \)
   (c) \( (A \cap B)^c \cup C \)
   (d) \( (B \cup C) \cap A^c \)

6. Define the following sets. Note that \( U \) is the universal set. Decide if each statement is true or false.
   \( U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\} \)
   \( A = \{1, 2, 3\} \)
   \( B = \{2, 4, 6\} \)
   \( C = \{3, 5, 7\} \)

   (a) \( B \text{ and } C \) are disjoint
   (b) \( 1 \subseteq A \)
   (c) \( B \subseteq B \)
   (d) \( \{3, 5\} \in C \)
   (e) \( A \cap B = 2 \)
   (f) \( A \cup C = \{1, 2, 3, 5, 7\} \)

\[
\begin{align*}
\text{1. } & (A \cap B)^c \text{ or } A^c \cup B^c \\
\text{2. } & A^c \cap B^c = \{ x | x \in A \text{ and } x \notin B \} \\
\text{3. } & \text{Turntable: } n(U) = 100, n(U \cap P) = 90, n(P) = 40 \\
& n(U \cap P) = n(U) + n(P) - n(U \cap P) \\
& 90 = 80 + 40 - n(U \cap P) \Rightarrow n(U \cap P) = 30 \\
& \text{only turntables: } n(U \cap P^c) = 50 \\
\text{4. } & 200 = a + b + c + d + e + f + g + h \\
& 55 = b + e \\
& 130 = a + b + c + h \\
& 30 = e \\
& 35 = b + c \\
& 55 = a + c + h \\
& 60 = b + d + f \\
& 15 = d
\end{align*}
\]
5) \[ \{ x | x \notin A \text{ or } x \in B \} \]

6) \( (A \cup B)^c = A^c \cap B^c \)

7) \((A \cap B^c) \cup C\)

8) \((B \cup C) \cap A^c\)

9) a) True – nothing in common
   b) False, \( 1 \in A \) and \( \{1\} \subseteq A \) are true
   c) False, \( B \subseteq B \) is true
   d) False, \( 3, 5 \in C \) and \( \{3, 5\} \subseteq C \) are true
   e) False, \( A \cap B = \{2\} \)
   f) False, \( A \cup C = \{1, 2, 3, 5, 7\} \)

\(\text{not two of these}\)
Exam 1 Review
Basic Probability

1. \( S = \{ g, s, b \} \), \( n(S) = 3 \) \( \Rightarrow \) 2\(^3\) = 8 events (subset of \( S \))

2. \( S = \{ w, o, o, d \} \) \( \Rightarrow \) \( n(S) = 4 \)

3. \[
\begin{array}{cccc}
1-1 & 2-1 & 3-1 & 4-1 \\
1-2 & 2-2 & 3-2 & 4-2 \\
1-3 & 2-3 & 3-3 & 4-3 \\
1-4 & 2-4 & 3-4 & 4-4 \\
1-5 & 2-5 & 3-5 & 4-5 \\
1-6 & 2-6 & 3-6 & 4-6 \\
\end{array}
\]

at least one 5 \( \quad P = \frac{12}{36} \)

sum > 10

4. Multiplying or coffee = 62% or .62

5a. \( E = \{ 10, 20 \} \) \( \Rightarrow \) \( P = \frac{2}{25} \)

b. \( E = \{ 2, 4, 5, 6, 8, 10, 12, 14, 15, 16, 18, 20, 22, 24, 25 \} \) \( P = \frac{15}{25} \)

c. \( E = \{ 3, 9, 15, 21 \} \) \( P = \frac{4}{25} \)

6. 52/52

7. \[
\begin{array}{cccc}
B & S & C & \text{Prob} \\
.3 & .11 & .04 & .07 \\
.24 & .09 & .05 & .02 \\
\end{array}
\]

a. \( P(B) = .57 \)

b. \( P(BNS) = .18 \)

c. \( P(BUP) = .74 \)

d. \( P(B \cap S \cap P^C) = .5 \)

e. \( P(BNS^C) = .39 \)

f. \( P(BNS) \cup (BNP) \cup (PNS)) = .39 \)

8. \[
\frac{P(E)}{P(E')} = \frac{P(E)}{1 - P(E)} = \frac{.55}{.45} = \frac{11}{9} \Rightarrow 11:9 \text{ odds} \\
\]

6:11 \( \Rightarrow \) \( P(F) = \frac{6}{6+11} = \frac{6}{17} \)
Exam Review
Conditional Probability

1. \( \begin{align*}
&\text{a) } P(S | P) = \frac{P(S \cap P)}{P(P)} = \frac{17 \times 9}{17 \times 9 + 13 \times 2} \\
&= \frac{1}{2} \\
&\text{b) } P(U | F) = \frac{P(U \cap F)}{P(F)} = \frac{13 \times 2}{17 \times 11 + 13 \times 8} \\
&= \frac{2}{11} \approx 0.18
\end{align*} \)

2. \( \begin{align*}
P(S1) &= (\frac{1}{4}) \times (\frac{1}{3}) \times (\frac{1}{2}) + (\frac{3}{4}) \times (\frac{1}{3}) \times (\frac{1}{2}) = \frac{1}{2} \\
P(S2) &= (\frac{1}{4}) \times (\frac{1}{3}) \times (\frac{1}{2}) + (\frac{3}{4}) \times (\frac{1}{3}) \times (\frac{1}{2}) = \frac{1}{4} \\
P(2 \text{ draws}) &= (\frac{1}{4}) \times (\frac{1}{3}) + (\frac{3}{4}) \times (\frac{1}{3}) = \frac{1}{4}
\end{align*} \)

3. \( \begin{align*}
&\text{a) } P(R | C) = \frac{n(R \cap C)}{P(C)} = \frac{5}{55} = \frac{1}{11} \\
&\text{b) NOT ME } \Rightarrow R \cap C \neq \emptyset \\
&P(R) \cdot P(C) = (\frac{4}{35})(\frac{15}{100}) = \frac{12}{35} \neq \frac{5}{7} = P(R \cap C) \\
&\Rightarrow \text{ NOT IND.}
\end{align*} \)

4. \( \begin{align*}
&\text{a) } P(P | NF) = \frac{n(P \cap NF)}{n(NF)} = \frac{5}{7} \\
&\text{b) } P(E) = P(F) = \frac{1}{6}, \frac{1}{3} = \frac{1}{6}
\end{align*} \)

5. \( \begin{align*}
&\text{a) Yes } P(E) = P(F) = \frac{1}{6}. \frac{1}{3} = \frac{1}{6} = P(E \cap F) \\
&\text{b) No. } P(F) \cap P(G) = \frac{1}{6} \neq \frac{1}{3} = P(F \cap G) \\
&\text{c) Yes } P(G) = 0 \\
&\text{d) } P(G | E) = \frac{n(G \cap E)}{n(E)} = 0 \\
&\text{e) } P(F | H) = \frac{n(F \cap H)}{n(H)} = \frac{1}{16}
\end{align*} \)
(b) \[ P(B_1|B_2) = \frac{P(B_1 \cap B_2)}{P(B_2)} = \frac{3/7 \cdot 5/6}{(3/7) \cdot 5/6 + 4/11 \cdot 9/11} \]

\[ = \frac{25}{57} \]

START:

- Box 1: 3B, 4R
- Box 2: 1B, 2R
- Box 3: 2B, 7R

Pick B₁:

- Box 2: 3B, 1R
- Box 3: 2B, 7R

Pick B₁, B₂:

- Box 1: 3B, 7R

Pick R₁, B₂:

- Box 2: 3B, 8R
- Box 3: 2B, B₁

Pick R₁, R₂:

- Box 3: 2B, 8R

C):

\[ P(B_3) = \frac{3/7 \cdot 5/6 \cdot 3/10}{(3/7) \cdot 5/6 + 4/11 \cdot 9/11} \]

\[ + \frac{4/11 \cdot 4/5 \cdot 3/11}{(4/11) \cdot 4/5 + 4/11 \cdot 1/5 \cdot 2/10} \]

\[ \approx 0.2705 \]