1.6 Conditional Probability

A survey is done of people making purchases at a gas station. Most people buy gas (Event A) or a drink (Event B).

	buy drink (B)	no drink (B^c)	total
buy gas (A)	45	25	70
no gas (A^c)	20	10	30
total	65	35	100

What is the probability that a person bought gas and a drink?

$$P(A \cap B) = 45/100 = \frac{n(A \cap B)}{n(S)}$$

What the probability that a person who buys a drink also buys gas? In other words, given that a person bought a drink (B), what is the probability that they bought gas (A)?

$$P = \frac{n(B \cap A)}{n(B)} = \frac{45}{65} = P(A \mid B)$$

Notation: $P(E \mid F)$ = the probability of E given F

The ***conditional probability*** of event E given event F is

$$P(E|F) = \frac{n(E \cap F)}{n(F)} = \frac{n(E \cap F)/n(S)}{n(F)/n(S)} = \frac{P(E \cap F)}{P(F)} = P(E|F)$$

What is the probability that a person who buys gas also buys a drink?

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{45/100}{70/100} = \frac{45}{70}$$

The Product Rule:

$P(E|F) = \frac{P(E \cap F)}{P(F)}$ $\cdot P(F) \Rightarrow P(E \cap F) = P(F) \cdot P(E|F)$

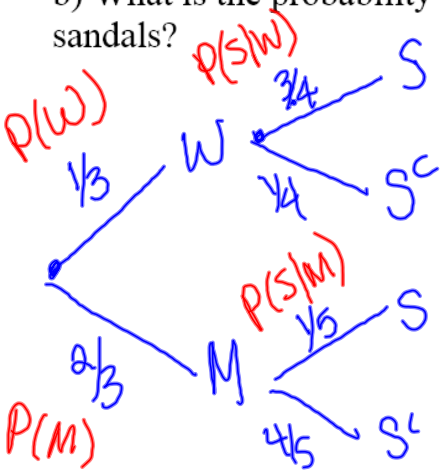
Example

At a party, 1/3 of the guests are women. 75% of the women wore sandals and 20% of the men wore sandals.

$P(W) = 1/3$ $P(S|W) = .75 = 3/4$ $P(S|M) = .2 = 1/5$

a) What is the probability that a person chosen at random at the party is a man wearing sandals? $P(M \cap S) = 2/15$

b) What is the probability that a ~~randomly chosen guest is wearing~~ sandals?



$P(W \cap S) = P(W) \cdot P(S|W) = \frac{1}{3} \cdot \frac{3}{4} = \frac{1}{4}$

$P(W \cap S^c) = \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$

$P(M \cap S) = \frac{2}{3} \cdot \frac{1}{5} = \frac{2}{15}$

$P(M \cap S^c) = \frac{2}{3} \cdot \frac{4}{5} = \frac{8}{15}$

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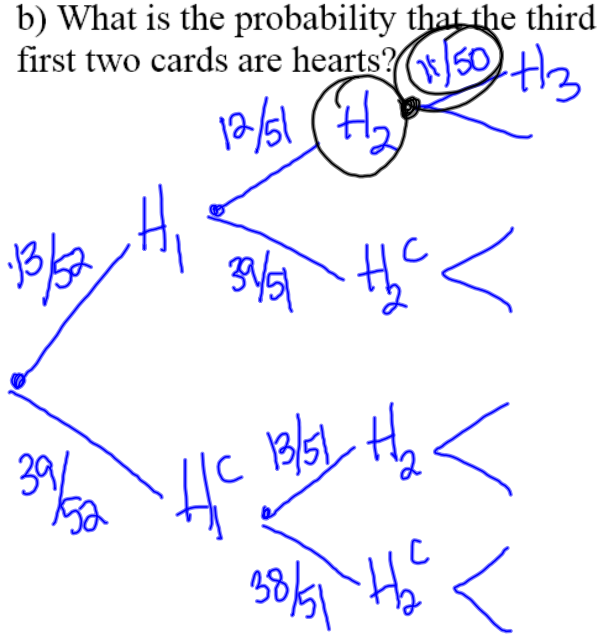
$P(S) = P(W \cap S) + P(M \cap S) = 1/4 + 2/15 = 23/60$

Example

Consider drawing 3 cards from a standard deck of 52 cards without replacement.

a) What is the probability that the three cards are hearts?

b) What is the probability that the third card drawn is a heart given the first two cards are hearts?



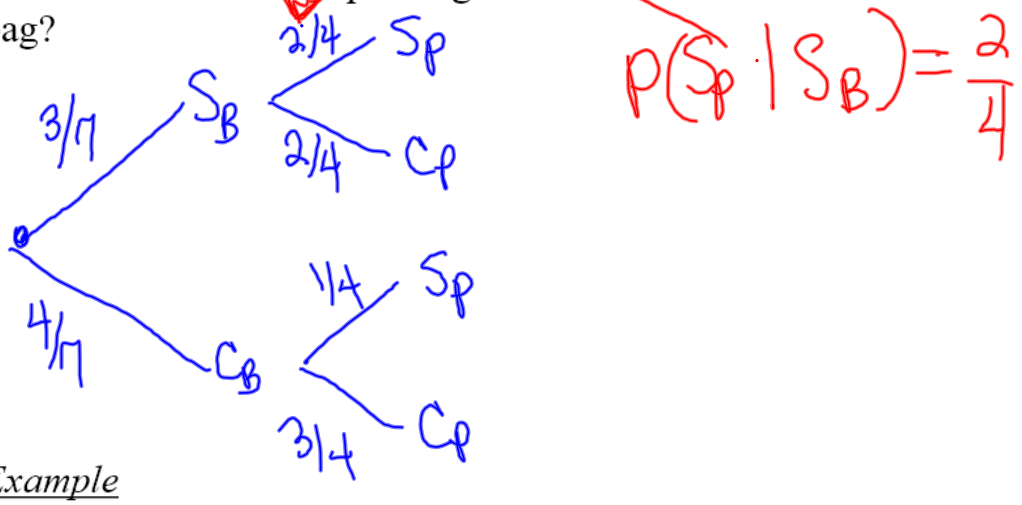
$P(H_1 \cap H_2 \cap H_3) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50}$

$= \frac{11}{850}$ $(\approx 1.3\%)$

$P(H_3 | H_1 \cap H_2) = \frac{11}{50}$

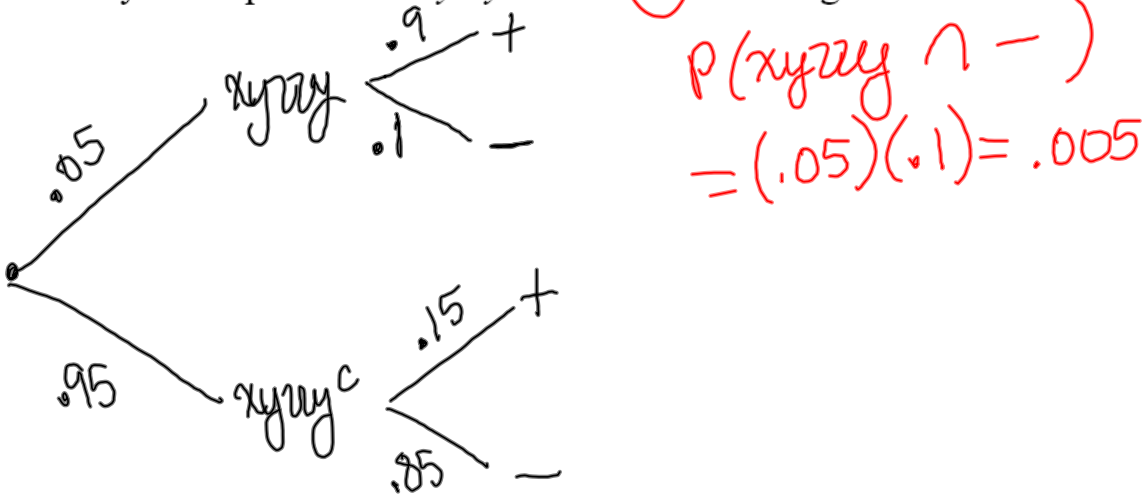
Example

A bag has 3 silver and 4 copper coins. A pouch has 1 silver and 2 copper coins. A coin is drawn at random from the bag and placed in the pouch. A coin is then drawn from the pouch. What is the probability that a silver coin is drawn from the pouch given that a silver coin was chosen from the bag?



Example

A medical test has been developed to detect *xyzzy* disease. It is estimated that 5% of the patients who come in for the test have the disease. When the test is given to a patient who has *xyzzy* disease, it is detected (positive) 90% of the time. When given to a patient who does not have *xyzzy* disease, a positive result is returned 15% of the time. What is the probability that a person has *xyzzy* disease and tests negative?



Independent Events: Events E and F are independent if $P(E \mid F) = P(E)$

$P(E|F) \overset{\text{if}}{=} P(E) = \frac{P(E \cap F)}{P(F)}$ $P(F)$

$P(E \cap F) = P(E) \cdot P(F)$ iff E and F are indep

Example

A medical experiment showed the probability that a new medicine was effective was 0.75, the probability of a certain side effect was 0.4 and the probability for both occurring is 0.3. Are these events independent?

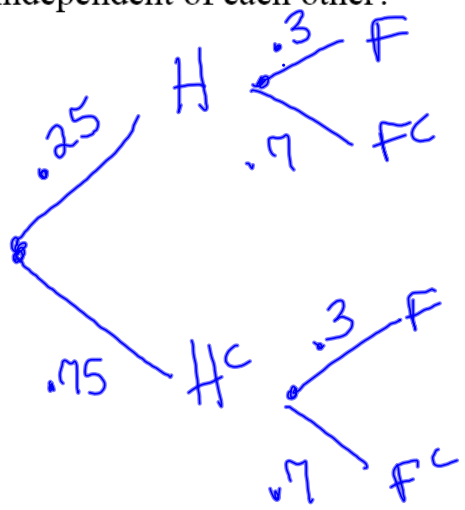
$P(E) = .75$, $P(S) = .4$, $P(E \cap S) = .3$

$P(E) \cdot P(S) = (.75)(.4) = .3 = P(E \cap S)$

$\Rightarrow$ INDEP

Example

The side effects of a certain medicine include a 25% chance of headaches and 30% chance of fatigue. What is the probability that a person taking this medicine will suffer exactly one of these side effects if they are independent of each other?



$P(H \cap F) = .25 \times .3$

$P(H \cap F^c) = (.25)(.7)$

+

$P(H^c \cap F) = (.75)(.3)$

.4

$P(H^c \cap F^c) = (.75)(.7)$

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Put your name, section and date on your paper.
Remove any loose bits of paper if it is torn from a spiral.

A bookshelf has 22 books on it. There are 7 are hardback books and 15 paperback books. There are 5 reference books (all hardbacks), 10 fiction books (all paperbacks) and 7 non-fiction books. What is the probability that a non-fiction book is a paperback?

1.7 Bayes' Theorem

Given $P(E \mid F)$, can we find $P(F \mid E)$?

Example
We are to choose a marble from a cup or a bowl. We need to flip a coin to decide to choose from the cup or the bowl. The bowl contains 1 red and 2 green marbles. The cup contains 3 red and 2 green marbles. What is the probability that a marble came from the bowl given that it is red?

Example
A survey of the local middle school found the percent of students in each grade who own a calculator. The results are below. What is the probability that a student with a calculator is in the 5^{th} grade?

Grade	4^{th}	5^{th}	6^{th}
Percent of student body	37	32	31
Percent that own a calculator	13	28	59

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Example

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