Exam 4 Learning Objectives

Voting Methods

➢ Given the results of an election, determine the winner using
  o Majority rule or plurality
  o Borda count
  o Hare method
  o Pairwise-comparison
  o Tournament method.
➢ Use the approval method to determine one or more winners in an election.
➢ Recognize when the following fairness criteria are violated
  o Majority Rule
  o Condorcet
  o Monotonicity
  o Irrelevant Alternatives

Chapter 13 - Fair Division

➢ Describe the goal of a fair-division problem and the terms equitable, envy-free, and Pareto-optimal.
➢ Divide a set of items using the adjusted winner procedure.
➢ Divide one or more items using the Knaster Inheritance Procedure.
➢ Divide a “cake” where the two or three players value parts of the cake differently.
Chapter 14 – Apportionment

➢ Explain the difference between quota and apportionment.
➢ State the quota condition and be able to tell which apportionment methods satisfy it and which do not.
➢ Know the paradoxes (Alabama, New State, and Population).
➢ Know that some methods have bias in favor of large or small states.
➢ Recognize the difference in computing quotas between the Hamilton method and divisor methods.
➢ Calculate the apportionment of seats in a representative body using the methods of
  o Hamilton
  o Jefferson
  o Adams
  o Webster
  o Hill-Huntington.
➢ Calculate the critical divisor for each state.
➢ Round a number using the geometric mean.
1. A union has 4 countries with the given populations. Use Hamilton’s method to apportion 99 seats. Then reapportion using 100 seats. Comment on the results. Munchkin (40,650), Quadling (38,650), Winkie (10,400), and Gillikin (10,300). Comment on your results.

2. A university teaches two languages, English and Klingon with the enrollments and Hamilton apportionment as shown

<table>
<thead>
<tr>
<th>Language</th>
<th># of students</th>
<th># of sections</th>
</tr>
</thead>
<tbody>
<tr>
<td>English</td>
<td>945</td>
<td>9</td>
</tr>
<tr>
<td>Klingon</td>
<td>9055</td>
<td>91</td>
</tr>
</tbody>
</table>

The school now wishes to offer Vulcan and the expected enrollment in Vulcan is 1040. Ten sections of language classes will be added. Reapportion using Hamilton’s method. Comment on your results.
3. Five people have chipped in money for 200 sunstones. The amount of money each person chipped in is: Anakin ($1320), Bobo ($1515), Chewie ($4935), Luke ($1118), and Yoda ($1112). Apportion the sunstones using Jefferson’s method. Comment on your results.

4. An instructor can teach 6 classes. Enrollment figures for three subjects are Introductory Potions (84), Intermediate Potions (34) and Advanced Potions (10). Determine the number of sections of each subject that will be offered using Webster’s method and the Hill-Huntington method.
5. Teachers are voting on where to go for their end-of-year field trip. The choices are Asgard (A), London (L), Narnia (N), or Pern (P). The results are:

\[\begin{array}{ccccc}
(\text{ALNP}) & (\text{LNPA}) & (\text{APLN}) & (\text{PLAN}) & (\text{LNAP}) \\
3 & 2 & 2 & 1 & 1 \\
\end{array}\]

Determine the location of the trip
(a) Using majority rule (apply the Hare method, if needed).
(b) Using a Borda count.

6. Students in a theater class are voting on which play to do for the Spring Show. The choices are Hamlet (H), Julius Caesar (J), King Lear (K) and Macbeth (M). The results are

\[\begin{array}{cccc}
(\text{HJKM}) & (\text{MKJH}) & (\text{KJMH}) & (\text{JMKH}) \\
12 & 14 & 7 & 6 \\
\end{array}\]

Determine which play will be done at the Spring Show using
(a) Pairwise comparison
(b) Tournaments with orders HJKM, KMJH, and HMJK
(c) Hare method
7. A town is voting for 3 council members using the approval method. From the table below, determine which three people will be on the council.

<table>
<thead>
<tr>
<th>Number of ballots</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidates</td>
</tr>
<tr>
<td>Athena</td>
</tr>
<tr>
<td>Hera</td>
</tr>
<tr>
<td>Poseidon</td>
</tr>
<tr>
<td>Zeus</td>
</tr>
<tr>
<td>Medusa</td>
</tr>
</tbody>
</table>

8. After having been roommates for four years at college, Sandy and Lucy are moving on. Several items they have accumulated belong jointly to the pair, but now must be divided between the two. They assign points to the items as follows:

<table>
<thead>
<tr>
<th>Object</th>
<th>Sandy's points</th>
<th>Lucy's points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bicycle</td>
<td>25</td>
<td>40</td>
</tr>
<tr>
<td>Barbells</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>Rowing machine</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>Game Console</td>
<td>15</td>
<td>18</td>
</tr>
<tr>
<td>Books</td>
<td>20</td>
<td>22</td>
</tr>
</tbody>
</table>

Use the adjusted winner procedure to determine a fair division of the property.
9. John, Ken, and Linda inherit a painting. If their monetary bids on the painting are $25,200, $21,600, and $18,000, respectively, what is the fair distribution arrived at by the Knaster inheritance procedure?

10. Bob and Carol view a cake as shown. They will use divide-and-choose to divide the cake between them. If Bob is the divider, how many units of value will Bob and Carol think he or she is receiving?

Bob's view 
Carol's view
11. Three players will share a cake with each player’s view of the cake as shown. If player 2 is the divider, who will end up with which pieces and how much cake does each play feel they have at the end of the procedure?

player 1

player 2

player 3

12. Suppose Kelly and Jan want to take turns, using the bottom-up strategy, to allocate several movie posters currently held jointly. Their ranked preferences are listed below:

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kelly</td>
<td>Batman</td>
<td>Spiderman</td>
<td>Harry Potter</td>
<td>Expendables</td>
</tr>
<tr>
<td>Jan</td>
<td>Spiderman</td>
<td>Harry Potter</td>
<td>Batman</td>
<td>Expendables</td>
</tr>
</tbody>
</table>

What is Kelly’s first pick if she uses a bottoms-up strategy and she chooses first?