Solutions to Self-Help Exercises 1.3

1. Consider the outcomes as ordered pairs, with the number on the bottom of the red one the first number and the number on the bottom of the white one the second number. The sample space is

$$\begin{aligned} S = \{ & (1,1),(1,2),(1,3),(1,4),\\ & (2,1),(2,2),(2,3),(2,4),\\ & (3,1),(3,2),(3,3),(3,4),\\ & (4,1),(4,2),(4,3),(4,4)\} \end{aligned}$$

2. **a.** $E = \{(1,1),(1,3),(3,1),(3,3)\}$, and $F = \{(1,4),(2,3),(3,2),(4,1)\}$

   b. $E \cap F = \emptyset$

   c. $E \cup F = \{(1,1),(1,3),(3,1),(3,3),(1,4),(2,3),(3,2),(4,1)\}$

   d. $G^c = \{(3,4),(4,3),(4,4)\}$

3. Since the baby can be any length greater than zero, the sample space is

$$S = \{x|x > 0, x \text{ in inches}\}$$

   a. $E = \{x|x > 22, x \text{ in inches}\}$

   b. $F = \{x|x \le 20, x \text{ in inches}\}$

   c. $G = \{x|19.5 < x < 21, x \text{ in inches}\}$

1.4 Basics of Probability

✧ Introduction to Probability

We first consider sample spaces for which the outcomes (elementary events) are equally likely. For example, a head or tail is equally likely to come up on a flip of a fair coin. Any of the six numbers on a fair die is equally likely to come up on a roll. We will refer to a sample space S whose individual elementary events are equally likely as a **uniform sample space**. We then give the following definition of the **probability** of any event in a uniform sample space.

Probability of an Event in a Uniform Sample Space

If S is a finite uniform sample space and E is any event, then the **probability of E**, $P(E)$, is given by

$$P(E) = \frac{\text{Number of elements in } E}{\text{Number of elements in } S} = \frac{n(E)}{n(S)}$$

EXAMPLE 1 **Probability for a Single Die** Suppose a fair die is rolled and the sample space is $S = \{1,2,3,4,5,6\}$. Determine the probability of each of the following events.
a. The die shows an odd number.
b. The die shows the number 9.
c. The die shows a number less than 8.

Solution **a.** We have $E = \{1,3,5\}$. Then

$$P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

b. The event F that the die shows a 9 is the impossible event. So $n(F) = 0$ and

$$P(F) = \frac{n(F)}{n(S)} = \frac{0}{6} = 0$$

c. The event G that the die shows a number less than 8 is just the certainty event. So $G = \{1,2,3,4,5,6\} = S$ and

$$P(G) = \frac{n(G)}{n(S)} = \frac{6}{6} = 1$$ ✦

HISTORICAL NOTE

The Beginnings of Probability

In 1654 the famous mathematician Blaise Pascal had a friend, Chevalier de Mere, a member of the French nobility and a gambler, who wanted to adjust gambling stakes so that he would be assured of winning if he played long enough. This gambler raised questions with Pascal such as the following: In eight throws of a die a player attempts to throw a one, but after three unsuccessful trials the game is interrupted. How should he be compensated? Pascal wrote to a leading mathematician of that day, Pierre de Fermat (1601–1665), about these problems, and their resulting correspondence represents the beginnings of the modern theory of mathematical probability.

EXAMPLE 2 **Probability for a Single Card** Suppose a single card is randomly drawn from a standard 52-card deck. Determine the probability of each of the following events.
a. A king is drawn.
b. A heart is drawn.

Solution **a.** The event is $E = \{$ K♢, K♡, K♠, K♣$\}$. So,

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

b. The event F contains 13 hearts. So

$$P(F) = \frac{n(F)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$ ✦

EXAMPLE 3 **Probability for Transistors** A bin contains 15 identical (to the eye) transistors except that 6 are defective and 9 are not. What is the probability that a transistor selected at random is defective?

Solution Let us denote the set S to be the set of all 15 transistors and the set E to be the set of defective transistors. Then,

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{15} = \frac{2}{5}$$ ✦

REMARK: What if we selected two transistors or two cards? We will learn how to handle this type of experiment in the next chapter.

EXAMPLE 4 **Probability for Two Coin Flips** A fair coin is flipped twice to observe whether heads or tails shows; order is important. What is the probability that tails occurs both times?

Solution The sample space S consists of the 4 outcomes

$$S = \{(H,H),(H,T),(T,H),(T,T)\}$$

Since we are using a fair coin, each of the individual four elementary events are equally likely. The set E that tails occurs both times is $E = \{(T, T)\}$ and contains one element. We have

$$P(E) = \frac{n(E)}{n(S)} = \frac{1}{4}$$ ✦

> ***HISTORICAL NOTE***
>
> **The Beginnings of Empirical Probability**
>
> Empirical probability began with the emergence of insurance companies. Insurance seems to have been originally used to protect merchant vessels and was in use even in Roman times. The first marine insurance companies began in Italy and Holland in the 14th century and spread to other countries by the 16th century. In fact, the famous Lloyd's of London was founded in the late 1600s. The first life insurance seems to have been written in the late 16th century in Europe. All of these companies naturally needed to know with what the likelihood of certain events would occur. The empirical probabilities were determined by collecting data over long periods of time.

✧ Empirical Probability

A very important type of problem that arises every day in business and science is to find a practical way to estimate the likelihood of certain events. For example, a food company may seek a practical method of estimating the likelihood that a new type of candy will be enjoyed by consumers. The most obvious procedure for the company to follow is to randomly select a consumer, have the consumer taste the candy, and then record the result. This should be repeated many times and the final totals tabulated to give the fraction of tested consumers who enjoy the candy. This fraction is then a practical estimate of the likelihood that all consumers will enjoy this candy. We refer to this fraction or number as **empirical probability**.

The London merchant John Graunt (1620–1674) with the publication of *Natural and Political Observations Made upon the Bills of Mortality* in 1662 seems to have been the first person to have gathered data on mortality rates and determined empirical probabilities from them. The data were extremely difficult to obtain. His then-famous London Life Table is reproduced below, showing the number of survivors through certain ages per 100 people.

Age	0	6	16	26	36	46	56	66	76
Survivors	100	64	40	25	16	10	6	3	1

London Life Table

EXAMPLE 5 Finding Empirical Probability Using the London Life Table, find the empirical probability of a randomly chosen person living in London in the first half of the 17th century surviving until age 46.

Solution In the London Life Table $N = 100$. If E is the event "survive to age 46," then according to the table the corresponding number is 10. Thus, the empirical probability of people living in London at that time surviving until age 46 was $10/100 = 0.1$. ✦

Consider now a poorly made die purchased at a discount store. Dice are made by drilling holes in the sides and then backfilling. Cheap dice are, of course, not carefully backfilled. So when a lot of holes are made in a face, such as for a side with 6, and they are not carefully backfilled, that side will not be quite as heavy as the others. Thus a 6 will tend to come up more often on the top. Even a die taken from a craps table in Las Vegas, where the dice are of very high quality, will have some tiny imbalance.

EXAMPLE 6 Finding Empirical Probability A die with 6 sides numbered from 1 to 6, such as used in the game of craps, is suspected to be somewhat lopsided. A laboratory has tossed this die 1000 times and obtained the results shown in the table. Find the empirical probability that a 2 will occur and the probability that a 6 will occur.

Outcome	1	2	3	4	5	6
Number Observed	161	179	148	177	210	125

Solution The total number observed is 1000. The number observed for the 2 and 6, respectively is 179 and 125. So dividing these numbers by 1000 gives

$$P(2) = 179/1000 = 0.179$$
$$P(6) = 125/1000 = 0.125$$

✦

CONNECTION
Frederick Mosteller and the Dice Experiment

Frederick Mosteller has been president of the American Association for the Advancement of Science, the Institute of Mathematical Statistics, and the American Statistical Association. He once decided that "It would be nice to see if the actual outcome of a real person tossing real dice would match up with the theory." He then engaged Willard H. Longcor to buy some dice, toss them, and keep careful records of the outcomes. Mr. Longcor then tossed the dice on his floor at home so that the dice would bounce on the floor and then up against the wall and then land back on the floor. After doing this several thousand times his wife became troubled by the noise. He then placed a rug on the floor and on the wall, and then proceeded to quietly toss his dice *millions* of times, keeping careful records of the outcomes. In fact, he was so careful and responsible about his task, that he threw away his data on the first 100,000 tosses, since he had a nagging worry that he might have made some mistake keeping perfect track.

✧ Probability Distribution Tables

A **probability distribution table** is a useful way to display probability data for an experiment. In a probability distribution table there is one column (or row) for the events that take place and one column (or row) for the probability of the event. The events chosen must be mutually exclusive and therefore the total probability will add to 1. This is best demonstrated through an example.

EXAMPLE 7 Flipping a Coin Twice Write the probability distribution table for the number of heads when a coin is flipped twice.

Solution Recall from Example 4 that the uniform sample space is $S = \{(H,H),(H,T),(T,H),(T,T)\}$. Next, we are asked to organize the events by the number of heads, so we will have three events

$E_1 = \{(H,H)\}$ with two heads and a probability of $1/4$,
$E_2 = \{(H,T),(T,H)\}$ with exactly one head and a probability of $2/4$
$E_3 = \{(T,T)\}$ with zero heads and a probability of $1/4$.
This is shown in the table on the left. ✦

Event	Probability
2 heads	$1/4$
1 head	$1/2$
0 heads	$1/4$

Note how the list of events covered all the possibilities for the number of heads and that the events are all mutually exclusive. You can't have exactly two heads and exactly one head at the same time! Next see that the sum of the probabilities is equal to one. This will always be the case when your probability distribution table is correct.

EXAMPLE 8 **Sum of the Numbers for Two Dice** Two fair dice are rolled. Find the probability distribution table for the sum of the numbers shown uppermost.

Solution Recall the uniform sample space in Example 2 of the last section for rolling two dice. We see the smallest sum is 2 from the roll (1,1) and the largest sum is 12 from the roll (6,6). Count the number of outcomes in each event to find the probability:

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

✦

EXAMPLE 9 **Weight of Oranges** A crate contains oranges and each orange is carefully weighed. It was found that 12 oranges weighed less than 100 grams, 40 oranges weighed 100 grams or more, but less than 150 grams, 60 oranges weighed 150 grams or more, but less than 200 grams, and 8 oranges weighed 200 grams or more. Organize this information in a probability distribution table

Solution The sample space for this experiment was found in the previous section to be $S = \{w|w > 0, w \text{ in grams}\}$. There are four mutually exclusive events described in this sample space and these form the basis of the probability distribution table. A total of $12 + 40 + 60 + 8 = 120$ oranges were weighed. The empirical probability that an orange weighs less than 100 grams will be the ratio $12/120$. The remaining probabilities are found in the same way. This gives the probability distribution table below where w is the weight of an orange in grams.

Event	Probability
$w < 100$	$12/120 = 1/10$
$100 \le w < 150$	$40/120 = 1/3$
$150 \le w < 200$	$60/120 = 1/2$
$w \ge 200$	$8/120 = 2/30$

✦

REMARK: Notice that in the probability distribution table above that there were no gaps and no overlap. It is important to be able to translate the statements like "100 grams or more" into an event $100 \le w$.

Self-Help Exercises 1.4

1. Two tetrahedrons (4 sided), each with equal sides numbered from 1 to 4, are identical except that one is red and the other white. If the two tetrahedrons are tossed and the number on the bottom face of each is observed, what is the sample space for this experiment? Write the probability distribution table for the sum of the numbers on the bottom of the dice.

2. In the past month 72 babies were born at a local hospital. Each baby was carefully measured and it was found that 10 babies were less than 19 inches long, 12 babies were 19 inches or longer but less than 20 inches long, 32 babies were 20 inches or longer but less than 21 inches long. Organize this information in a probability distribution table.

3. An experiment consists of randomly selecting a letter from the word FINITE and observing it. What is the probability of selecting a vowel?

1.4 Exercises

In Exercises 1 through 4, a fair die is tossed. Find the probabilities of the given events.

1. an even number

2. the numbers 4 or 5

3. a number less than 5

4. any number except 2 or 5

In Exercises 5 through 10, a card is drawn randomly from a standard deck of 52 cards. Find the probabilities of the given events.

5. an ace

6. a spade

7. a red card

8. any number between 3 and 5 inclusive

9. any black card between 5 and 7 inclusive

10. a red 8

In Exercises 11 through 14, a basket contains 3 white, 4 yellow, and 5 black transistors. If a transistor is randomly picked, find the probability of each of the given events.

11. white

12. black

13. not yellow

14. black or yellow

15. A somewhat lopsided die is tossed 1000 times with 1 showing on the top face 150 times. What is the empirical probability that a 1 will show?

16. A coin is flipped 10,000 times with heads showing 5050 times. What is the empirical probability that heads will show?

17. The speed of 500 vehicles on a highway with limit of 55 mph was observed, with 400 going between 55 and 65 mph, 60 going less than 55 mph, and 40 going over 65 mph. What is the empirical probability that a vehicle chosen at random on this highway will be going **a.** under 55 mph, **b.** between 55 and 65 mph, **c.** over 65 mph.

18. In a survey of 1000 randomly selected consumers, 50 said they bought brand A cereal, 60 said they bought brand B, and 80 said they bought brand C. What is the empirical probability that a consumer will purchase **a.** brand A cereal, **b.** brand B, **c.** brand C?

19. A large dose of a suspected carcinogen has been given to 500 white rats in a laboratory experiment. During the next year, 280 rats get cancer. What is the empirical probability that a rat chosen randomly from this group of 500 will get cancer?

20. A new brand of sausage is tested on 200 randomly selected customers in grocery stores with 40 saying they like the product, the others saying they do not. What is the empirical probability that a consumer will like this brand of sausage?

21. Over a number of years the grade distribution in a mathematics course was observed to be

A	B	C	D	F
25	35	80	40	20

What is the empirical probability that a randomly selected student taking this course will receive a grade of A? B? C? D? F?

22. A store sells four different brands of VCRs. During the past year the following number of sales of each of the brands were found.

Brand A	Brand B	Brand C	Brand D
20	60	100	70

What is the empirical probability that a randomly selected customer who buys a VCR at this store will pick brand A’? brand B:’ brand C’? brand D?

23. A somewhat lopsided die is tossed 1000 times with the following results. What is the empirical probability that an even number shows?

1	2	3	4	5	6
150	200	140	250	160	100

24. A retail store that sells sneakers notes the following number of sneakers of each size that were sold last year.

7	8	9	10	11	12
20	40	60	30	40	10

What is the empirical probability that a customer buys a pair of sneakers of size 7 or 12?

25. A fair coin is flipped three times, and heads or tails is observed after each flip. What is the probability of the event "at least 2 heads are observed." Refer to the answer in Exercise 3 in Section 4.3.

26. A fair coin is flipped, and it is noted whether heads or tails show. A fair die is tossed, and the number on the top face is noted. What is the probability of the event "heads shows on the coin and an even number on the die." Refer to the answer in Exercise 4 in Section 4.3.

27. A coin is flipped three times. If heads show, one is written down. If tails show, zero is written down. What is the probability of the event "one is observed at least twice." Refer to the answer in Exercise 5 in Section 4.3.

28. Two fair tetrahedrons (4 sided), each with equal sides numbered from 1 to 4, are identical except that one is red and the other white. If the two tetrahedrons are tossed and the number on the bottom face of each is observed, what is the probability of the event "the sum of the numbers is 4." Refer to the answer in Exercise 6 in Section 4.3.

In Exercises 29 through 34, assume that all elementary events in the same sample space are equally likely.

29. A fair coin is flipped three times. What is the probability of obtaining exactly 2 heads? At least 1 head?

30. A family has three children. Assuming a boy is as likely as a girl to have been born, what is the probability that two are boys and one is a girl? That at least one is a boy?

31. A fair coin is flipped and a fair die is tossed. What is the probability of obtaining a head and a 3?

32. A fair coin is flipped twice and a fair die is tossed. What is the probability of obtaining 2 heads and a 3?

33. A pair of fair dice are tossed. What is the probability of obtaining a sum of 2? 4? 8?

34. A pair of fair dice are tossed. What is the probability of obtaining a sum of 5? 6? ll?

35. An experiment consists of selecting a digit from the number 112964333 and observing it. What is the probability that "an even digit is selected."

36. An experiment consists of selecting a letter from the word CONNECTICUT and observing it. What is the probability that "a vowel is selected."

Solutions to Self-Help Exercises 1.4

1. The sample space was found in the previous section and is

$$\begin{aligned} S = \{ & (1,1),(1,2),(1,3),(1,4), \\ & (2,1),(2,2),(2,3),(2,4), \\ & (3,1),(3,2),(3,3),(3,4), \\ & (4,1),(4,2),(4,3),(4,4)\} \end{aligned}$$

The sum of the numbers ranges from $1+1=2$ to $4+4=8$. Count the outcomes in each event to find

Sum	2	3	4	5	6	7	8
Probability	1/16	2/16	3/16	4/16	3/16	2/16	1/16

2. The sample space for this experiment is $S = \{x|x > 0, x \text{ in inches}\}$. The lengths of $10+12+32 = 54$ babies is given. A careful examination of the events shows that no mention was made of babies longer than 21 inches. We deduce that $72-54 = 18$ babies must be 21 inches or longer. This can now be arranged in a probability distribution table where x is the length of the baby in inches.

Event	Probability
$x < 19$	$10/72 = 5/36$
$19 \le x < 20$	$12/72 = 1/6$
$20 \le x < 21$	$32/72 = 4/9$
$x \ge 21$	$18/72 = 1/4$

3. FINITE has six letters and there are three vowels. So

$$P(\text{vowel}) = \frac{3}{6} = \frac{1}{2}$$

1.5 Rules for Probability

✧ Elementary Rules

Recall that if S is a finite uniform sample space, that is, a space for which all individual elementary elements are equally likely, and E is any event, then the probability of E, denoted by $P(E)$, is given by

$$P(E) = \frac{\text{Number of elements in } E}{\text{Number of elements in } S} = \frac{n(E)}{n(S)}$$

If E is an event in a sample space then $0 \le n(E) \le n(S)$. Dividing this by $n(S)$ then gives

$$0 \le \frac{n(E)}{n(S)} \le \frac{n(S)}{n(S)} = 1$$

Using the definition of probability given above yields

$$0 \le P(E) \le 1$$

This is our first rule for probability. Notice also that

$$P(S) = \frac{n(S)}{n(S)} = 1 \quad \text{and} \quad P(\emptyset) = \frac{n(\emptyset)}{n(S)} = \frac{0}{n(S)} = 0$$

These rules apply for events in any sample space; however, the derivations just given are valid only for spaces with equally likely events.

Elementary Rules for Probability

For any event E in a sample space S we have

$$0 \le P(E) \le 1 \qquad P(S) = 1 \qquad P(\emptyset) = 0$$

> **HISTORICAL NOTE**
>
> **Some Developments in Probability**
>
> Neither Pascal nor Fermat published their initial findings on probability. Christian Huygens (1629–1695) became acquainted with the work of Pascal and Fermat and subsequently published in 1657 the first tract on probability: *On Reasoning in Games of Dice*. This little pamphlet remained the only published work on probability for the next 50 years. James Bernoulli (1654–1705) published the first substantial tract on probability when his *Art of Reasoning* appeared 7 years after his death. This expanded considerably on Huygens' work. The next major milestone in probability occurred with the publication in 1718 of Abraham De Moivre's work *Doctrine of Chance: A Method of Calculating of Events in Play*. Before 1770, probability was almost entirely restricted to the study of gambling and actuarial problems, although some applications in errors of observation, population, and certain political and social phenomena had been touched on. It was Pierre Simon Laplace (1739–1827) who broadened the mathematical treatment of probability beyond games of chance to many areas of scientific research. The theory of probability undoubtedly owes more to Laplace than to any other individual.

✧ Union Rule for Probability

We would now like to determine the probability of the union of two events E and F. We start by recalling the union rule for sets:

$$n(E \cup F) = n(E) + n(F) - n(E \cap F)$$

Now divide both sides of this last equation by $n(S)$ and obtain

$$\frac{n(E \cup F)}{n(S)} = \frac{n(E)}{n(S)} + \frac{n(F)}{n(S)} - \frac{n(E \cap F)}{n(S)}$$

By the definition of probability given above this becomes

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

This is called the **union rule for probability**. This rule applies for events in any sample space; however, the derivation just given is valid only for spaces with equally likely events.

> **Union Rule for Probability**
>
> $$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

EXAMPLE 1 **Union Rule With Drawing a Card** A single card is randomly drawn from a standard deck of cards. What is the probability that it will be a red card or a king.

Solution Let R be the set of red cards and let K be the set of kings. Red cards consist of hearts and diamonds, so there are 26 red cards. Therefore $P(R) = 26/52$. There are 4 kings, so $P(K) = 4/52$. Among the 4 kings, there are 2 red cards. So, $P(R \cap K) = 2/52$. Using the union rule gives

$$\begin{aligned} P(R \cup K) &= P(R) + P(K) - P(R \cap K) \\ &= \frac{26}{52} + \frac{4}{52} - \frac{2}{52} \\ &= \frac{28}{52} = \frac{7}{13} \end{aligned}$$

✦

REMARK: It is likely that you would intuitively use the union rule had you been asked to pick out all of the cards from the deck that were red or kings. You would choose out all of the red cards along with all of the kings for a total of 28 cards.

EXAMPLE 2 **Union Rule With Two Dice** Two dice, identical except that one is green and the other is red, are tossed and the number of dots on the top face of each is observed. Let E consist of those outcomes for which the number of dots on the top face of the green dice is a 1 or 2. Let F be the event that the sum of the number of dots on the top faces of the two dice is 6. Find the probability that a 1 or 2 will be on the top of the green die or the sum of the two numbers will be 6.

E	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
F	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Figure 1.21

Solution Notice that

$$\begin{aligned} E &= \{(1,1),(1,2),\cdots,(1,6),(2,1),(2,2),\cdots,(2,6)\} \\ F &= \{(1,5),(2,4),(3,3),(4,2),(5,1)\} \\ E\cap F &= \{(1,5),(2,4)\} \end{aligned}$$

The set that a 1 or 2 will be on the top of the green die and the sum of the two numbers will be 6 is $E\cap F$. To find $p(E\cap F)$ use the union rule of probability and obtain

$$\begin{aligned} P(E\cup F) &= P(E)+P(F)-P(E\cap F) \\ &= \frac{12}{36}+\frac{5}{36}-\frac{2}{36}=\frac{15}{36} \end{aligned}$$

Alternatively, you can draw the sample space for two dice and circle all outcomes that have a 1 or 2 on the top of the green die or the sum of the two numbers shown uppermost is 6. This is done in Figure 1.21. Counting the circled outcomes we find there are 15 of them. ✦

Consider two events E and F that are mutually exclusive, that is, $E\cap F)=\emptyset$. Then $P(E\cap F)=0$. Using the union rule of probability for these two sets gives

$$\begin{aligned} P(E\cup F) &= P(E)+P(F)-P(E\cap F) \\ &= P(E)+P(F)-0 \\ &= P(E)+P(F) \end{aligned}$$

We then have the following rule:

Union Rule for Mutually Exclusive Events
If E and F are mutually exclusive events, then

$$P(E\cup F)=P(E)+P(F)$$

For any event E in a sample space, $E\cup E^c=S$ and $E\cap E^c=\emptyset$. So, E and E^c are mutually exclusive. Using the union rule for mutually exclusive events we have that

$$P(E)+P(E^c)=P(E\cup E^c)=P(S)=1$$

So, $P(E^c)=1-P(E)$ and $P(E)=1-P(E^c)$. We call this the **complement rule**.

Complement Rule for Probability

$$P(E^c)=1-P(E) \qquad P(E)=1-P(E^c)$$

EXAMPLE 3 **Complement Rule for Two Dice** Consider the dice described in Example 2. What is the probability that the sum of the two numbers is less than 12.

Solution Let E be the event that the sum of the two numbers is less than 12. Then we wish to find $P(E)$. It is tedious to find this directly. Notice that $E^c = \{(6,6)\}$. Now use the complement rule.

$$P(E) = 1 - P(E^c) = 1 - \frac{1}{36} = \frac{35}{36}$$ ✦

EXAMPLE 4 **Finding Empirical Probability** A die with 6 sides numbered from 1 to 6, such as used in the game of craps, is suspected to be somewhat lopsided. A laboratory has tossed this die 1000 times and obtained the results shown in the table. Find the empirical probability that an even number will occur.

Outcome	1	2	3	4	5	6
Number Observed	161	179	148	177	210	125

Solution The total number observed is 1000. The number observed for the 2, 4, and 6, respectively is 179, 177, and 125. So dividing these numbers by 1000 gives

$$P(2) = 179/1000 = 0.179$$
$$P(4) = 177/1000 = 0.177$$
$$P(6) = 125/1000 = 0.125$$

To find the empirical probability of an even number these three values can be added as the events are mutually exclusive. That is,

$$P(\text{even}) = P(2) + P(4) + P(6) = 0.179 + 0.177 + 0.125 = 0.481$$ ✦

EXAMPLE 5 **Finding the Probability of an Event** A salesman makes two stops when in Pittsburgh. The first stop yields a sale 10% of the time, the second stop 15% of the time, and both stops yield a sale 4% of the time. What proportion of the time does a trip to Pittsburgh result in no sales?

Solution Let E be the event a sale is made at the first stop and F the event that a sale is made at the second stop. What should we make of the statement that the first stop yields a sale 10% of the time. It seems reasonable to assume that the salesman or his manager have looked at his sales data and estimated the 10% number. We then take the 10% or 0.10 as the empirical probability. We interpret the other percentages in a similar way. We then have

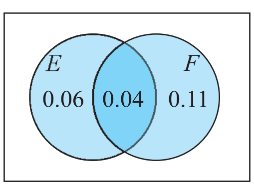


Figure 1.22

$$P(E) = 0.10 \qquad P(F) = 0.15 \qquad P(E \cap F) = 0.04$$

Since $P(E \cap F) = 0.04$, we place 0.04 in the region $E \cap F$ in Figure 1.22. Now since $P(E) = 0.10$, we can see that $P(E \cap F^c) = 0.10 - 0.04 = 0.06$. In a similar fashion we have $P(E^c \cap F) = 0.15 - 0.04 = 0.11$. Thus, we readily see from Figure 1.22 that

$$P(E \cup F) = 0.06 + 0.04 + 0.11 = 0.21$$

Then by the complement rule we have

$$P((E \cup F)^c) = 1 - P(E \cup F) = 1 - 0.21 = 0.79$$

Thus no sale is made in Pittsburgh 79% of the time. ✦

REMARK: We could have obtained $P(E \cup F)$ directly from the union rule as follows:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F) = 0.10 + 0.15 - 0.04 = 0.21$$

EXAMPLE 6 Finding the Probability of an Event The probability that any of the first five numbers of a loaded die will come up is the same while the probability that a 6 comes up is 0.25. What is the probability that a 1 will come up?

Solution We are given $P(1) = P(2) = P(3) = P(4) = P(5)$, $P(6) = 0.25$. Also, all the probabilities must add up to 1, so

$$\begin{aligned} 1 &= P(1) + P(2) + P(3) + P(4) + P(5) + P(6) \\ &= 5P(1) + 0.25 \\ 5P(1) &= 0.75 \\ P(1) &= 0.15 \end{aligned}$$

✦

EXAMPLE 7 Continuous Sample Space Arrange the following information in a probability distribution table: A crop of apples is brought in for weighing. It is found that 10% of the apples weigh less than 100 gm, 40% weigh 200 gm or less, and 25% weigh more than 300 gm.

Solution If we let x = weight of an apple in grams then

Event	Probability
$0 \leq x < 100$ gm	0.10
100 gm $\leq x \leq$ 200 gm	0.30
200 gm $< x \leq$ 300 gm	0.35
$x > 300$ gm	0.25

Note that the 40% of the apples that weigh 200 gm or less includes the 10% that weigh less than 100 grams. Since the events in a probability distribution table must be mutually exclusive, the 30% that weigh 100 grams or more and 200 grams or less are shown in the second row. The third row of the table is found using deductive reasoning as the total probability must be 1 and there is a gap in the events. ✦

✧ Odds (Optional)

One can interpret probabilities in terms of **odds** in a bet. Suppose in a sample space S we are given an event E with probability $P = P(E) = \frac{5}{7}$. In the long term we expect E to occur 5 out of 7 times. Now, $P(E^c) = \frac{2}{7}$ and in the long term we expect that E^c to occur 2 out of 7 times. Then we say that the **odds in favor** of E are 5 to 2.

Odds

The **odds in favor** of an event E are defined to be the ratio of $P(E)$ to $P(E^c)$, or

$$\frac{P(E)}{P(E^c)} = \frac{P(E)}{1-P(E)}$$

Often the ratio ${P(E)}/{P(E^c)}$ is reduced to lowest terms, a/b, and then we say that **the odds are a to b** or a:b.

EXAMPLE 8 Determining the Odds of an Event You believe that a horse has a probability of $1/4$ of winning a race. What are the odds of this horse winning? What are the odds of this horse losing? What profit should a winning \$2 bet return to be fair?

Solution Since the probability of winning is $P = 1/4$, the odds of winning are

$$\frac{P}{1-P} = \frac{1/4}{1-1/4} = \frac{1/4}{3/4} = \frac{1}{3}$$

that is: 1 to 3 or 1:3.

Since the probability of winning is $\frac{1}{4}$, the probability of losing is $1 - 1/4 = 3/4$. Then the odds for losing is

$$\frac{3/4}{1-3/4} = \frac{3/4}{1/4} = \frac{3}{1}$$

or 3 to 1 or 3:1. Since the fraction $3/1$ can also be written as $6/2$ with odds 6 to 2, a fair \$2 bet should return \$6 for a winning ticket. ✦

Notice that making this same bet many times, we expect to win \$6 one-fourth of the time and lose \$2 three-fourths of the time. So, for example, on every four bets we would expect to win \$6 once and lose \$2 three times. Our average winnings would be $6(1) - 2(3) = 0$ dollars.

If the odds for an event E are given as a/b, we can calculate the probability $P(E)$. We have

$$\begin{aligned}
\frac{a}{b} &= \frac{P(E)}{1-P(E)} \\
a(1-P(E)) &= bP(E) \\
a &= bP(E) + aP(E) \\
&= P(E)(a+b) \\
P(E) &= \frac{a}{a+b}
\end{aligned}$$

Obtaining Probability From Odds

Suppose that the odds for an event E occurring is given as a/b or $a:b$, then

$$P(E) = \frac{a}{a+b}$$

REMARK: One can think of the odds a:b of event E as saying if this experiment was carried out $a+b$ times, then a of those times E would have occurred. Our definition of empirical probability then says $P(E) = \frac{a}{a+b}$

EXAMPLE 9 **Obtaining Probability From Odds** At the race track, the odds for a horse winning is listed at $3/2$. What is the probability that the horse will win.

Solution Using the above formula for odds a/b, we have

$$P = \frac{a}{a+b} = \frac{3}{3+2} = 0.60$$

✦

Self-Help Exercises 1.5

1. If $S = \{a,b,c\}$ with $P(a) = P(b) = 2P(c)$, find $P(a)$.
2. A company has bids on two contracts. They believe that the probability of obtaining the first contract is 0.4 and of obtaining the second contract is 0.3, while the probability of obtaining both contracts is 0.1.
   a. Find the probability that they will obtain exactly one of the contracts.
   b. Find the probability that they will obtain neither of the contracts.
3. What are the odds that the company in the previous exercise will obtain both of the contracts?

1.5 Exercises

In all the following, S is assumed to be a sample space.

1. Let $S = \{a,b,c\}$ with $P(a) = 0.1$, $P(b) = 0.4$, and $P(c) = 0.5$. Let $E = \{a,b\}$ and $F = \{b,c\}$. Find $P(E)$ and $P(F)$.
2. Let $S = \{a,b,c,d,e,f\}$ with $P(a) = 0.1$, $P(b) = 0.2$, $P(c) = 0.25$, $P(d) = 0.15$, $P(e) = 0.12$, and $P(f) = 0.18$. Let $E = \{a,b,c\}$ and $F = \{c,d,e,f\}$ and find $P(E)$ and $P(F)$.
3. Let $S = \{a,b,c,d,e,f\}$ with $P(b) = 0.2$, $P(c) = 0.25$, $P(d) = 0.15$, $P(e) = 0.12$, and $P(f) = 0.1$. Let $E = \{a,b,c\}$ and $F = \{c,d,e,f\}$. Find $P(a)$, $P(E)$, and $P(F)$.
4. Let $S = \{a,b,c,d,e,f\}$ with $P(b) = 0.3$, $P(c) = 0.15$, $P(d) = 0.05$, $P(e) = 0.2$, $P(f) = 0.13$. Let $E = \{a,b,c\}$ and $F = \{c,d,e,f\}$. Find $P(a)$, $P(E)$, and $P(F)$.
5. If $S = \{a,b,c,d\}$ with $P(a) = P(b) = P(c) = P(d)$, find $P(a)$.
6. If $S = \{a,b,c\}$ with $P(a) = P(b)$ and $P(c) = 0.4$, find $P(a)$.
7. If $S = \{a,b,c,d,e,f\}$ with $P(a) = P(b) = P(c) = P(d) = P(e) = P(f)$, find $P(a)$.
8. If $S = \{a,b,c\}$ with $P(a) = 2P(b) = 3P(c)$, find $P(a)$.
9. If $S = \{a,b,c,d,e,f\}$ with $P(a) = P(b) = P(c)$, $P(d) = P(e) = P(f) = 0.1$, find $P(a)$.
10. If $S = \{a,b,c,d,e,f\}$ and if $P(a) = P(b) = P(c)$, $P(d) = P(e) = P(f)$, $P(d) = 2P(a)$, find $P(a)$.
11. If E and F are two disjoint events in S with $P(E) = 0.2$ and $P(F) = 0.4$, find $P(E \cup F)$, $P(E^c)$, and $P(E \cap F)$.
12. Why is it not possible for E and F to be two disjoint events in S with $P(E) = 0.5$ and $P(F) = 0.7$?
13. If E and F are two disjoint events in S with $P(E) = 0.4$ and $P(F) = 0.3$, find $P(E \cup F)$, $P(F^c)$, $P(E \cap F)$, $P((E \cup F)^c)$, and $P((E \cap F)^c)$.
14. Why is it not possible for $S = \{a,b,c\}$ with $P(a) = 0.3$, $P(b) = 0.4$, and $P(c) = 0.5$?

15. Let E and F be two events in S with $P(E) = 0.3$, $P(F) = 0.5$, and $P(E \cap F) = 0.2$. Find $P(E \cap F)$ and $P(E \cap F^c)$.

16. Let E and F be two events in S with $P(E) = 0.3$, $P(F) = 0.5$, and $P(E \cap F) = 0.2$. Find $P(E^c \cap F)$ and $P(E^c \cap F^c)$.

17. Let E and F be two events in S with $P(E) = 0.3$, $P(F) = 0.5$, and $P(E \cup F) = 0.6$. Find $P(E \cap F)$ and $P(E \cap F^c)$.

18. Why is it not possible to have E and F two events in S with $P(E) = 0.3$ and $P(E \cap F) = 0.5$?

In Exercises 19 through 22, let E, F, and G be events in S with $P(E) = 0.55$, $P(F) = 0.4$, $P(G) = 0.45$, $P(E \cap F) = 0.3$, $P(E \cap G) = 0.2$, $P(F \cap G) = 0.15$, and $P(E \cap F \cap G) = 0.1$.

19. Find $P(E \cap F \cap G^c)$, $P(E \cap F^c \cap G)$, and $P(E \cap F^c \cap G^c)$.

20. Using the results of the previous exercise, find $P(E^c \cap F \cap G)$, $P(E^c \cap F \cap G^c)$, and $P(E^c \cap F^c \cap G)$.

21. Using the results of the previous two exercises, find $P(E \cup F \cup G)$.

22. Using the results of the previous three exercises, find $P(E^c \cup F^c \cup G^c)$.

23. For the loaded die in Example 6 of the text, what are the odds that

a. a 2 will occur **b.** a 6 will occur?

24. For the loaded die in Example 6 of the text, what are the odds that

a. a 3 will occur **b.** a 1 will occur?

25. A company believes it has a probability of 0.40 of receiving a contract. What is the odds that it will?

26. In Example 5 of the text, what are the odds that the salesman will make a sale on

a. the first stop **b.** on the second stop

c. on both stops?

27. It is known that the odds that E will occur are 1:3 and that the odds that F will occur are 1:2, and that both E and F cannot occur. What are the odds that E or F will occur?

28. If the odds for a successful marriage are 1:2, what is the probability for a successful marriage?

29. If the odds for the Giants winning the World Series are 1:4, what is the probability that the Giants will win the Series?

Applications

30. Bidding on Contracts An aerospace firm has three bids on government contracts and knows that the contracts are most likely to be divided up among a number of companies. The firm decides that the probability of obtaining exactly one contract is 0.6, of exactly two contracts is 0.15, and of exactly three contracts is 0.04. What is the probability that the firm will obtain at least one contracts? No contracts?

31. Quality Control An inspection of computers manufactured at a plant reveals that 2% of the monitors are defective, 3% of the keyboards are defective, and 1% of the computers have both defects.

a. Find the probability that a computer at this plant has at least one of these defects.

b. Find the probability that a computer at this plant has none of these defects.

32. Medicine A new medication produces headaches in 5% of the users, upset stomach in 15%, and both in 2%.

a. Find the probability that at least one of these side effects occurs.

b. Find the probability that neither of these side effects occurs.

33. Manufacturing A manufactured item is guaranteed for one year and has three critical parts. It has been decided that during the first year the probability of failure of the first part is 0.03, of the second part 0.02, the third part 0.01, both the first and second is 0.005, both the first and third is 0.004, both the second and third is 0.003, and all three parts 0.001.

a. What is the probability that exactly one of these parts will fail in the first year?

b. What is the probability that at least one of these parts will fail in the first year?

c. What is the probability that none of these parts will fail in the first year?

34. Marketing A survey of business executives found that 40% read *Business Week*, 50% read *Fortune*, 40% read *Money*, 17% read both *Business Week* and *Fortune*, 15% read both both *Business Week* and *Money*, 14% read both *Fortune* and *Money*, and 8% read all three of these magazines.

a. What is the probability that one of these executives reads exactly one of these three magazines?

b. What is the probability that one of these executives reads at least one of these three magazines?

c. What is the probability that one of these executives reads none of these three magazines?

35. Advertising A firm advertises three different products, A, B, and C, on television. From past experience, it expects 1.5% of listeners to buy exactly one of the products, 1% to buy exactly two of the products, 1.2% to buy A, 0.4% to buy both A and B, 0.3% to buy both A and C, and 0.6% to buy A but not the other two.

a. Find the probability that a listener will buy only B or only C.

b. Find the probability that a listener will buy all three.

c. Find the probability that a listener will buy both B and C.

d. Find the probability that a listener will buy none of the three.

36. Sales A salesman always makes a sale at one of the three stops in Atlanta and 30% of the time makes a sale at only the first stop, 15% at only the second stop, 20% at only the third stop, and 35% of the time at exactly two of the stops. Find the probability that the salesman makes a sale at all three stops in Atlanta.

Extensions

37. Let E and F be two events in S with $P(E) = 0.5$ and $P(F) = 0.7$. Just how small could $P(E \cap F)$ possibly be?

38. Let E and F be two events in S with $P(E) = 0.3$ and $P(F) = 0.4$. Just how large could $P(F \cup F)$ possibly be?

39. You buy a new die and toss it 1000 times. A 1 comes up 165 times. Is it true that the probability of a 1 showing on this die is 0.165?

40. A fair coin is to be tossed 100 times. Naturally you expect tails to come up 50 times. After 60 tosses, heads has come up 40 times. Is it true that now heads is likely to come up less often than tails during the next 40 tosses?

41. You are playing a game at a casino and have correctly calculated the probability of winning any one game to be 0.48. You have played for some time and have won 60% of the time. You are on a roll. Should you keep playing?

42. You are watching a roulette game at a casino and notice that red has been coming up unusually often. (Red should come up as often as black.) Is it true that, according to "the law of averages," black is likely to come up unusually often in the next number of games to "even things up"?

43. You buy a die and toss it 1000 times and notice that a 1 came up 165 times. You decide that the probability of a 1 on this die is 0.165. Your friend takes this die and tosses it 1000 times and notes that a 1 came up 170 times. He concludes that the probability of a 1 is 0.17. Who is correct?

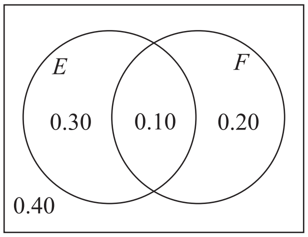
44. People who frequent casinos and play lotteries are gamblers, but those who run the casinos and lotteries are not. Do you agree? Why or why not?

Solutions to Self-Help Exercises 1.5

1. If $S = \{a, b, c\}$ and $P(a) = P(b) = 2P(c)$, then

$$\begin{aligned} 1 &= P(a) + P(b) + P(c) \\ &= P(a) + P(a) + 0.5P(a) \\ &= 2.5P(a) \\ P(a) &= 0.4 \end{aligned}$$

2. a. Let E be the event that the company obtains the first contract and let F be the event that the company obtains the second contract. The event that the company obtains the first contract but not the second is $E \cap F^c$, while



the event that the company obtains the second contract but not the first is $E^c \cap F$. These two sets are mutually exclusive, so the probability that the company receives exactly one of the contracts is

$$P(E \cap F^c) + P(E^c \cap F)$$

Now $P(E) = 0.40$, $P(F) = 0.30$, and since $E \cap F$ is the event that the company receives both contracts, $P(E \cap F) = 0.10$. Notice on the accompanying diagram that $E \cap F^c$ and $E \cap F$ are mutually disjoint and that $(E \cap F^c) \cup (E \cap F) = E$. Thus

$$\begin{aligned} P(E \cap F^c) + P(E \cap F) &= P(E) \\ P(E \cap F^c) + 0.10 &= 0.40 \\ P(E \cap F^c) &= 0.30 \end{aligned}$$

Also notice on the accompanying diagram that $E^c \cap F$ and $E \cap F$ are mutually disjoint and that $(E^c \cap F) \cup (E \cap F) = F$. Thus

$$\begin{aligned} P(E^c \cap F) + P(E \cap F) &= P(F) \\ P(E^c \cap F) + 0.10 &= 0.30 \\ P(E \cap F^c) &= 0.20 \end{aligned}$$

Thus the probability that the company will receive exactly one of the contracts is

$$P(E \cap F^c) + P(E^c \cap F) = 0.30 + 0.20 = 0.50$$

b. The event that the company obtains neither contract is given by $(E \cup F)^c$. From the diagram

$$P(E \cup F) = 0.30 + 0.10 + 0.20 = 0.60$$

Thus

$$P((E \cup F)^c) = 1 - P(E \cup F) = 1 - 0.60 = 0.40$$

The probability that the company receives neither contract is 0.40.

1.6 Conditional Probability

APPLICATION
Locating Defective Parts

A company makes the components for a product at a central location. These components are shipped to three plants, 1 , 2, and 3, for assembly into a final product. The percentages of the product assembled by the three plants are, respectively, 50%, 20%, and 30%. The percentages of defective products coming from these three plants are, respectively, 1%, 2%, and 3%. What is the probability of randomly choosing a product made by this company that is defective from Plant 1? See Example 4 for the answer.

1.4 EXERCISES

1. 1/2 **3.** 2/3 **5.** 1/13 **7.** 1/2
9. 3/26 **11.** 1/4 **13.** 2/3 **15.** 0.15
17. **a.** 0.12 **b.** 0.8 **c.** 0.08 **19.** 0.56
21. A: 0.125, B: 0.175, C: 0.4, D: 0.2, F: 0.1
23. 0.55 **25.** 1/2 **27.** 1/2 **29.** 3/8, 7/8
31. 1/12 **33.** 1/36, 1/12, 5/36 **35.** 1/3

1.5 EXERCISES

1. 0.50, 0.90 **3.** 0.18, 0.63, 0.62 **5.** 0.25
7. 1/6 **9.** 7/30 **11.** 0.60, 0.80, 0
13. 0.70, 0.70, 0, 0.30, 1 **15.** 0.60, 0.10
17. 0.20, 0.10 **19.** 0.20, 0.10, 0.15
21. 0.85 **23.** **a.** 3:17 **b.** 1:3
25. 2:3 **27.** 7:5 **29.** 0.20
31. 0.04, 0.96 **33.** 0.039, 0.049, 0.951
35. **a.** 0.009 **b.** 0.001 **c.** 0.006 **d.** 0.974
37. 0.20
39. You do not know what the actual probability is. You do know that the empirical probability is $165/1000 = 0.165$. This represents the best guess for the actual probability. But if you tossed the coin more times, the relative frequency and the new empirical probability would most likely had changed.
41. The probabilities in the game are constant and do not change just because you are on a winning streak. Thus no matter what has happened to you in the past, the probability of winning any one game remains constant at 0.48. Thus if you continue to play, you should expect to win 48% of the time in the future. You have been lucky to have won 60% of the time up until now.
43. After reading the first discussion problem above, we know that it is, in fact, impossible to determine with certainty the actual probability precisely. Since the die has been tossed a total of 2000 times and a one has come up 335 times, our best guess at the probability is $335/2000 = 0.1675$.

1.6 EXERCISES

1. 3/7, 3/5 **3.** 2/3, 4/5 **5.** 1, 0
7. 2/3, 1/2 **9.** 1/3, 1 **11.** 0, 1/2
13. No **15.** Yes **17.** No
19. Yes **21.** 2/11 **23.** 1/7
25. **a.** $\frac{10,200}{132,600} \approx 0.077$ **b.** $\frac{49}{25\cdot 33} \approx 0.059$
27. 0.12, 0.64, 0.60 **29.** 1/3, 3/10
31. No **33.** 0.65 **35.** 0.72
37. 0.02, 0.017 **39.** 0.026
41. 0.000001 **43.** 5/7, 5/21, 1/21
45. Yes **47.** No **49.** 0.057818
51. For E and F to be independent, they must satisfy $P(E)\times P(F) = P(E\cap F)$. From the Venn diagram, we must have: $(p_1+p_2)\times(p_3+p_2) = p_2$. So,

$$p_1p_3 + p_2p_3 + p_1p_2 + p_2^2 = p_2$$

$$p_1p_3 = p_2(1-p_3-p_2-p_1) = p_2p_4$$

The above steps can be reversed, so if $p_1p_3 = p_2p_4$, we will have $P(E)\times P(F) = P(E\cap F)$.

If the sets are mutually disjoint, then $p_2 = 0$. This implies that $p_1p_3 = p_2p_4 = 0$. Then either p_1 or p_3 or both are zero. Thus either $P(E) = 0$, $P(F) = 0$, or both.
53. Since E and F are independent, $P(E)\times P(F) = P(E\cap F)$.

$$\begin{aligned} P(E^c\cap F^c) &= 1-P(E\cup F)\\ &= 1-\big(P(E)+P(F)-P(E\cap F)\big)\\ &= 1-P(E)-P(F)+P(E)\times P(F)\\ &= \big(1-P(E)\big)\times\big(1-P(F)\big)\\ &= P(E^c)\times P(F^c) \end{aligned}$$

Hence, if E and F are independent, so are E^c and F^c.
55. Since E and F are exclusive, $P(E\cap F) = P(\emptyset) = 0$. Since $P(E)$ and $P(F)$ are both nonzero, then $P(E)\times P(F) > 0$. Therefore, E and F are not independent.
57.

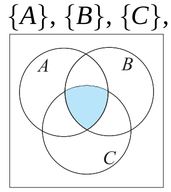
$$\begin{aligned} P(E^c|F) &= \frac{P(E^c\cap F)}{P(F)}\\ &= \frac{P(F)-P(E\cap F)}{P(F)}\\ &= 1-P(E|F) \end{aligned}$$

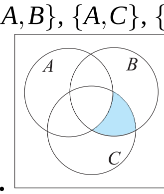
59. $P(E|F) = \frac{P(E\cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$
61. $P(E|F)+P(E^c|F) = \frac{P(E\cap F)+P(E^c\cap F)}{P(F)} = \frac{P(F)}{P(F)} = 1$

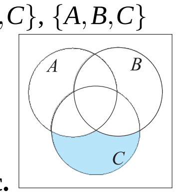
1.7 EXERCISES

1. 3/7, 57/93 **3.** 3/19, 1/3 **5.** 4/11
7. **a.** 2/23 **b.** 21/23 **9.** 2/17
11. **a.** 1/56 **b.** 5/28 **13.** 2/7
15. 4/19 **17.** 13/51 **19.** 13/51
21. 6/17 **23.** $100/136 \approx 0.74$
25. $396/937 \approx 0.42$, $5/937 \approx 0.005$
27. 5/7 **29.** 10/19 **31.** **a.** 1/12 **b.** 1/4
33. $P(1|N) = 6/20$, $P(2|N) = 4/20$, $P(3|N) = 6/20$, $P(4|N) = 4/20$

REVIEW EXERCISES

1. **a.** Yes **b.** No **c.** Yes
2. $\{x \mid x = 5n, n$ is an integer and $1 \le n \le 8\}$
3. $\{0, -\sqrt{2}, \sqrt{2}\}$
4. $\emptyset$, $\{A\}$, $\{B\}$, $\{C\}$, $\{A,B\}$, $\{A,C\}$, $\{B,C\}$, $\{A,B,C\}$
5. **a.**


b.


c.


6. $A\cup B = \{1,2,3,4\}$, $A\cap B = \{2,3\}$, $B^c = \{1,5,6\}$, $A\cap B\cap C = \{2,3\}\cap\{4,5\} = \emptyset$, $(A\cup B)\cap C = \{4\}$, $A\cap B^c\cap C = \emptyset$
7. **a.** My current instructors who are less than 6 feet tall. **b.** My current instructors who are at least 6 feet tall or are male. **c.** My current instructors who are female and weigh at most 180 pounds. **d.** My current male instructors who are at least 6 feet tall and weigh more than 180 pounds. **e.** My current male instructors who are less than 6 feet tall and weigh more