Applied Finite Mathematics

Tomastik/Epstein

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Applied Finite Mathematics, Second Edition

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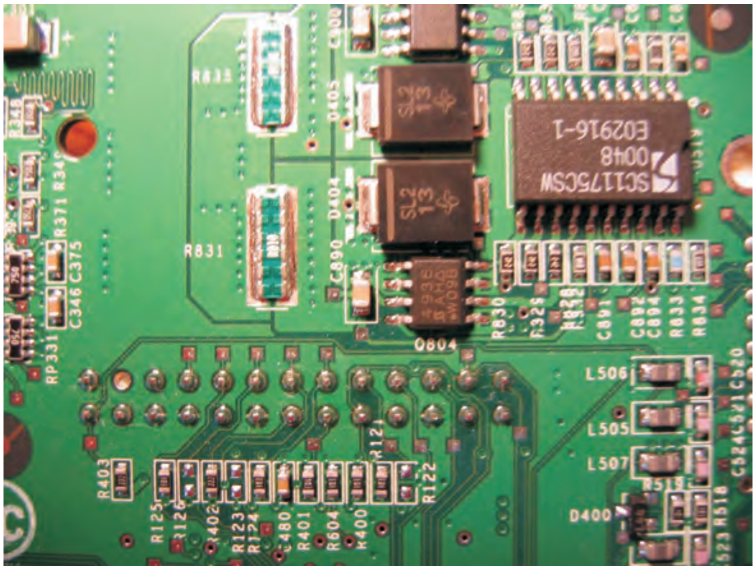
Logic



CONNECTION

Circuit Boards

How should the circuits on this board be laid out so that the video card works? Logic is used in the design of circuit boards.

L.1 Introduction to Logic

✧ Statements

Logic is the science of correct reasoning and of making valid conclusions. In logic conclusions must be inescapable. Every concept must be clearly defined. Thus, dictionary definitions are often not sufficient since there can be no ambiguities or vagueness.

We restrict our study to declarative sentences that are unambiguous and that can be classified as true or false but not both. Such declarative sentences are called **statements** and form the basis of logic.

> **Statements**
> A **statement** is a declarative sentence that is either true or false but not both.

Thus, commands, questions, exclamations, and ambiguous sentences cannot be statements.

> ***HISTORICAL NOTE***
>
> **A Brief History of Logic**
>
> The Greek philosopher Aristotle (384–322 B.C.) is generally given the credit for the first systemic study of logic. His work, however, used ordinary language. The second great period of logic came with Gottfried Leibnitz (1646–1716), who initiated the use of symbols to simplify complicated logical arguments. This treatment is referred to as **symbolic logic** or **mathematical logic**. In symbolic logic, symbols and prescribed rules are used very much as in ordinary algebra. This frees the subject from the ambiguities of ordinary language and permits the subject to proceed and develop in a methodical way. It was, however, Augustus De Morgan (1806–1871) and George Boole (1815–1864) who systemically developed symbolic logic. The “algebra” of logic that they developed removed logic from philosophy and attached it to mathematics.

EXAMPLE 1 **Determining if Sentences Are Statements** Decide which of the following sentences are statements and which are not.

a. Look at me.
b. Do you enjoy music?
c. What a beautiful sunset!
d. Two plus two equals four.
e. Two plus two equals five.
f. The author got out of bed after 6:00 A.M. today.
g. That was a great game.
h. $x+2=5$.

Solution The first three sentences are not statements since the first is a command, the second is a question, and the third is an exclamation. Sentences **d** and **e** are statements; **d** is a true statement while **e** is a false statement. Sentence **f** is a statement, but you do not know if it is true or not. Sentence **g** is not a statement since we are not told what “great” means. With a definition of great, such as “Our team won,” then it would be a statement. The last sentence **h.** is not a statement since it cannot be classified as true or false. For example, if $x=3$ it is true. But if $x=2$ it is false. ✦

✧ Connectives

A statement such as “I have money in my pocket” is called a **simple** statement since it expresses a single thought. But we need to also deal with **compound** statements such as “I have money in my pocket and my gas tank is full.” We will let letters such as p, q, and r denote simple statements. To write compound statements, we need to introduce symbols for the **connectives**.

Connectives
A **connective** is a word or words, such as "and" or "if and only if," that is used to combine two or more simple statements into a compound statement.

We will consider the 5 connectives given in the following table. We will discuss the first 3 in this section and the last 2 in the third section of this chapter.

Name	Connective	Symbol
Conjunction	and	$\wedge$
Disjunction	or	$\vee$
Negation	not	$\sim$
Conditional	if ... then	$\rightarrow$
Biconditional	if and only if	$\leftrightarrow$

Logic does not concern itself with whether a simple statement is true or false. But if all the simple statements that make up a compound statement are known to be true or false, then the rules of logic will enable us to determine if the compound statement is true or false. We will do this in the next section.

We now carefully give the definitions of the three connectives "and," "or," and "not." Notice that the precise meanings of the three compound statements that involve these connectives are incomplete unless a clear statement is made as to when the compound statement is true and when it is false. The first connective we discuss is **conjunction** which is the concept of "and."

Conjunction
A **conjunction** is a statement of the form "p and q" and is written symbolically as

$$p \wedge q$$

The conjunction $p \wedge q$ is true if both p and q are true, otherwise it is false.

EXAMPLE 2 **Using Conjunction** Write the compound statement "I have money in my pocket and my gas tank is full" in symbolic form.

Solution First let p be the statement "I have money in my pocket" and q be the statement "my gas tank is full." Since $\wedge$ represents the word "and," the compound statement can be written symbolically as $p \wedge q$. ✦

The next connective we consider is **disjunction** which is the concept of "or." Make careful note of the fact that the logical "or" is slightly different in meaning than the typical English use of the word "or."

Disjunction

A **disjunction** is a statement of the form “p or q” and is written symbolically as

$$p \vee q$$

The disjunction $p \vee q$ is false if both p and q are false and is true in all other cases.

REMARK: The word “or” in this definition conveys the meaning “one or the other, or both.” This is also called the **inclusive or**.

EXAMPLE 3 **Using Disjunction** Write the compound statement “Janet is in the top 10% of her class or she lives on campus” in symbolic form.

Solution First let p be the statement “Janet is in the top 10% of her class” and q the statement “She lives on campus.” Since $\vee$ represents the word “or,” the compound statement can be written as $p \vee q$. ✦

REMARK: In everyday language the word “or” is not always used in the way indicated above. For example, if a car salesman tells you that for $20,000 you can have a new car with automatic transmission or a new car with air conditioning, he means “one or the other, but not both.” This use of the word “or” is called **exclusive or**.

The final connective introduced in this section is **negation** which is the concept of “not.”

Negation

Negation

A **negation** is a statement of the form “not p” and is written symbolically as

$$\sim p$$

The negation $\sim p$ is true if p is false and false if p is true.

For example, if p is the statement “Janet is smart,” then $\sim p$ is the statement “Janet is not smart.”

EXAMPLE 4 **Using Negation** Let p and q be the following statements:

p: George Bush plays football for the Washington Redskins.
q: The Dow Jones industrial average set a new record high last week.

Write the following statements in symbolic form.

a. George Bush does not play football for the Washington Redskins, and the Dow Jones industrial average set a new record high last week.

b. George Bush plays football for the Washington Redskins, or the Dow Jones industrial average did not set a new record high last week.

c. George Bush does not play football for the Washington Redskins, and the Dow Jones industrial average did not set a new record high last week.

d. It is not true that George Bush plays football for the Washington Redskins and that the Dow Jones industrial average set a new record high last week.

Solution **a.** $(\sim p)\wedge q$ **b.** $p\vee\sim q$ **c.** $\sim p\wedge\sim q$ **d.** $\sim(p\wedge q)$ ✦

EXAMPLE 5 **Translating Symbolic Forms Into Compound Statements** Let p and q be the following statements:

p: Philadelphia is the capital of New Jersey.
q: General Electric lost money last year.

Write out the statements that correspond to each of the following:
a. $p\vee q$ **b.** $p\wedge q$ **c.** $p\vee\sim q$ **d.** $\sim p\wedge\sim q$

Solution

a. Philadelphia is the capital of New Jersey, or General Electric lost money last year.

b. Philadelphia is the capital of New Jersey, and General Electric lost money last year.

c. Philadelphia is the capital of New Jersey, or General Electric did not lose money last year.

d. Philadelphia is not the capital of New Jersey, and General Electric did not lose money last year.

✦

In most cases when dealing with complex compound statements, there will not be a question as to the order in which to apply the connectives. However, you may have noticed in the above examples that negation was used before disjunction or conjunction. The order of precedence for logical connectives is stated below.

Order of Precedence
The logical connectives are used in the following order

$$\sim, \wedge, \vee, \rightarrow, \leftrightarrow$$

Self-Help Exercises L.1

1. Determine which of the following sentences are statements:

a. The Atlanta Braves won the World Series in 1992.

b. IBM makes oil tankers for Denmark.

c. Does IBM make oil tankers for Denmark?

d. Please pay attention.

e. I have a three-dollar bill in my purse, or I don't have a purse.

2. Let p be the statement "George Washington was never president of the United States" and q be the statement "George Washington wore a wig." Write out the statements that correspond to the following:

a. $\sim p$ **b.** $p \vee q$ **c.** $\sim p \wedge q$
d. $p \wedge \sim q$ **e.** $\sim p \vee \sim q$

L.1 Exercises

In Exercises 1 through 14, decide which are statements.

1. Water freezes at 70^oF.

2. It rained in St. Louis on May 4, 1992.

3. $5 > 10$.

4. This sentence is false.

5. The number 4 is not a prime.

6. How are you feeling?

7. I feel great!

8. $10 + 10 - 5 = 25$

9. There is life on Mars.

10. Cleveland is the largest city in Ohio.

11. Who said Cleveland is the largest city in Ohio?

12. You don't say!

13. IBM lost money in 1947.

14. Groundhog Day is on February 12.

15. Let p and q denote the following statements:

p: George Washington was the third president of the United States.
q: Austin is the capital of Texas.

Express the following compound statements in words:

a. $\sim p$ **b.** $p \wedge q$ **c.** $p \vee q$
d. $\sim p \wedge q$ **e.** $p \vee \sim q$ **f.** $\sim (p \wedge q)$

16. Let p and q denote the following statements:

p: Mount McKinley is the highest point in the United States.
q: George Washington was a signer of the Declaration of Independence.

Express the following compound statements in words.

a. $\sim q$ **b.** $p \wedge q$ **c.** $p \vee q$
d. $p \wedge \sim q$ **e.** $\sim p \wedge \sim q$ **f.** $\sim (p \vee q)$

17. Let p and q denote the following statements:

p: George Washington owned over 100,000 acres of property.
q: The Exxon Valdez was a luxury liner.

a. State the negation of these statements in words.
b. State the disjunction for these statements in words.
c. State the conjunction for these statements in words.

18. Let p and q denote the following statements:

p: McDonald's Corporation operates large farms.
q: Wendy's Corporation operates fast-food restaurants.

a. State the negation of these statements in words.
b. State the disjunction for these statements in words.
c. State the conjunction for these statements in words.

19. Let p and q denote the following statements:

p: The *Wall Street Journal* has the highest daily circulation of any newspaper.
q: *Advise and Consent* was written by Irving Stone.

Give a symbolic expression for the statements below.

a. *Advise and Consent* was not written by Irving Stone.
b. The *Wall Street Journal* has the highest daily circulation of any newspaper, and *Advise and Consent* was not written by Irving Stone.
c. The *Wall Street Journal* has the highest daily circulation of any newspaper, or *Advise and Consent* was written by Irving Stone.

d. The *Wall Street Journal* does not have the highest daily circulation of any newspaper, or *Advise and Consent* was not written by Irving Stone.

20. Let p and q denote the following statements:

p: IBM makes computers.

q: IBM makes trucks.

Give a symbolic expression for the statements below.

a. IBM does not make trucks.

b. IBM makes computers, or IBM makes trucks.

c. IBM makes computers, or IBM does not make trucks.

d. IBM does not make computers, and IBM does not make trucks.

Solutions to Self-Help Exercises L.1

1. The sentences **a**, **b**, and **e** are statements, while **c** and **d** are not.

2. **a.** George Washington was a president of the United States.

b. George Washington was never president of the United States, or George Washington wore a wig.

c. George Washington was a president of the United States, and George Washington wore a wig.

d. George Washington was never president of the United States, and George Washington did not wear a wig.

e. George Washington was a president of the United States, or George Washington did not wear a wig.

L.2 Truth Tables

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Table L.1

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table L.2

p	$\sim p$
T	F
F	T

Table L.3

p	q	$\sim q$	$p\vee \sim q$
T	T	F	T
T	F	T	T
F	T	F	F
F	F	T	T

Table L.4

✧ Introduction to Truth Tables

The **truth value** of a statement is either true or false. Thus the statement "Ronald H. Coarse won the Nobel Prize in Economics in 1991" has truth value true since it is a true statement, whereas the statement "Los Angeles is the capital of California" has truth value false since it is a false statement.

Logic does not concern itself with the truth value of simple statements. But if we know the truth values of the simple statements that make up a compound statement, then logic can determine the truth value of the compound statement.

For example, to understand the very definition of $p \vee q$, one must know under what conditions the compound statement will be true. As defined in the last section $p \vee q$ is always true unless both p and q are false. A convenient way of summarizing this is by a truth table. This is done in Table L.1.

The truth tables for the statements $p \wedge q$ and $\sim p$ are given in Table L.2 and Table L.3. As Table L.2 indicates, $p \wedge q$ is true only if both p and q are true. Given a general compound statement, we wish to determine the truth value given any possible combination of truth values for the simple statements that are contained in the compound statement. We use a truth table for this purpose. The next examples illustrate how this is done.

EXAMPLE 1 **Constructing a Truth Table** Construct a truth table for the statement $p\vee \sim q$.

Solution Place p and q at the head of the first two columns as indicated in Table L.4 $\vee$ found in Table L.1. ✦ and list all possible truth values for p and q as indicated. It is strongly recommended that you always list the truth values in the first two columns in the same way. This will be particularly useful later when we will need to compare two truth tables. Now enter the truth values for $\sim q$ in the third column. Now using the first and third columns of the table, construct the fourth column using the definition of

EXAMPLE 2 **Constructing a Truth Table** Construct a truth table for the statement $\sim p \wedge (p \vee q)$.

Solution Make the same first two columns as before. Next make a column for $\sim p$ and the corresponding truth values. Now make a fourth column for $p \vee q$. Finally, using the third and fourth columns and the definition of $\wedge$, fill in the fifth column of Table L.5.

p	q	$\sim p$	$p \vee q$	$\sim p \wedge (p \vee q)$
T	T	F	T	F
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

Table L.5

Thus we see that $\sim p \wedge (p \vee q)$ is true only if p is false and q is true. ✦

We can construct a truth table for a compound statement with three simple statements.

EXAMPLE 3 **Constructing a Truth Table** Construct a truth table for the statement $(p \wedge q) \wedge [(r \vee \sim p) \wedge q]$.

Solution Always use the same order of T's and F's that are indicated in the first three columns of Table L.6. Fill in the rest of the columns in the order given.

p	q	r	$p \wedge q$	$\sim p$	$r \vee \sim p$	$(r \vee \sim p) \wedge q$	$(p \wedge q) \wedge [(r \vee \sim p) \wedge q]$
T	T	T	T	F	T	T	T
T	T	F	T	F	F	F	F
T	F	T	F	F	T	F	F
T	F	F	F	F	F	F	F
F	T	T	F	T	T	T	F
F	T	F	F	T	T	T	F
F	F	T	F	T	T	F	F
F	F	F	F	T	T	F	F

Table L.6

We see that $(p \wedge q) \wedge [(r \vee \sim p) \wedge q]$ is true only if p, q, and r are all true. ✦

✧ Exclusive Disjunction

p	q	$p \underline{\vee} q$
T	T	F
T	F	T
F	T	T
F	F	F

Table L.7

We now consider the exclusive "or." Recall that the exclusive "or" means "one or the other, but not both." The truth table for the exclusive disjunction is given in Table L.7 where we note that the symbol for the exclusive disjunction is $\underline{\vee}$. Notice that $\underline{\vee}$ is true only if exactly one of the two statements is true.

REMARK: Unless clearly specified otherwise, the word "or" will always be taken in the *in*clusive sense.

EXAMPLE 4 **Determining the Truth Value of a Statement** Let p and q be the following statements:

p: Aaron Copland was an American composer.
q: Rudolf Serkin was a violinist.

Determine the truth value of each of the following statements:

a. $p \vee q$ **b.** $\sim (p \vee q)$ **c.** $p \underline{\vee} q$ **d.** $\sim (p \underline{\vee} q)$ **e.** $p \wedge \sim q$

Solution First note that p is true and q is false.[1] Both the disjunction in **a** and the exclusive disjunction in **c** are therefore true. Thus their negations in **b** and **d** are false. The statement in **e** is the conjunction of a true statement p with a true statement $\sim q$ and thus is true. ✦

✧ Tautology and Contradiction

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

Table L.8

The statement $p \wedge \sim p$ is always false according to the truth table in Table L.8. In such a case, we say that the statement $p \wedge \sim p$ is a **contradiction**. If the statement is always true, we say that the statement is a **tautology**.

[1] Serkin was a famous pianist.

> **Contradiction and Tautology**
> We say that a statement is a **contradiction** if the truth value of the statement is always false no matter what the truth values of its simple component statements. We say that a statement is a **tautology** if the truth value of the statement is always true no matter what the truth values of its simple component statements.

EXAMPLE 5 **Determining if a Statement Is a Tautology** Determine if the statement $p \wedge (\sim p \wedge q)$ is a tautology.

Solution Create a truth table.

p	q	$\sim p$	$\sim p \vee q$	$p \wedge (\sim p \vee q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

The truth table indicates that the statement is true no matter what the truth values of p and q are. Thus, this statement is a tautology. ✦

Self-Help Exercises L.2

1. Construct the truth table for the statement

$$(p \vee q) \vee (r \wedge \sim q)$$

2. Let p be the statement "George Washington was the first president of the United States" and q be the statement "George Washington wore a wig." Determine the truth value of each of the statements below.

a. $\sim p$ **b.** $p \vee q$ **c.** $\sim p \wedge q$
d. $p \wedge \sim q$ **e.** $\sim p \vee \sim q$

L.2 Exercises

In Exercises 1 through 20, construct a truth table for the given statement. Indicate if a statement is a tautology or a contradiction.

1. $p \wedge \sim q$ **2.** $p \underline{\vee} \sim q$

3. $\sim (\sim p)$ **4.** $\sim (p \wedge q)$

5. $(p \wedge \sim q) \vee q$ **6.** $(p \underline{\vee} q) \vee \sim q$

7. $\sim p \vee (p \wedge q)$ **8.** $\sim p \underline{\vee} (p \wedge q)$

9. $(p \vee q) \wedge (p \wedge q)$ **10.** $(p \wedge q) \vee (p \vee q)$

11. $(p \vee \sim q) \vee (\sim p \wedge q)$ **12.** $(p \wedge \sim q) \wedge (p \vee \sim q)$

13. $(p \vee q) \wedge r$ **14.** $p \vee (q \wedge r)$

15. $\sim [(p \wedge q) \wedge r]$ **16.** $\sim [p \wedge (q \wedge r)]$

17. $(p \vee q) \vee (q \wedge r)$ **18.** $(p \wedge q) \wedge (q \vee r)$

19. $(p \vee \sim q) \vee (\sim q \wedge r)$ **20.** $(\sim p \wedge q) \wedge (\sim q \underline{\vee} r)$

21. Let p and q be the statements:

p: Roe v. Wade was a famous boxing match.

q: Iraq invaded Kuwait in 1990.

Determine the truth value of the following compound statements:

a. $\sim p$ **b.** $p \wedge q$ **c.** $p \vee q$
d. $\sim p \wedge q$ **e.** $p \vee \sim q$

Note that p is false and q is true.

22. Let p and q be the statements:

p: The sun rises in the east.

q: Proctor & Gamble is a casino in Las Vegas.

Determine the truth value of the following compound statements:

a. $\sim q$ **b.** $p \wedge q$ **c.** $p \underline{\vee} q$
d. $p \vee \sim q$ **e.** $\sim p \vee q$

23. Let p and q be the statements:

p: The South Pole is the southernmost point on the Earth.

q: The North Pole is a monument in Washington, D.C.

Determine the truth value of the following compound statements:

a. $\sim q$ **b.** $p \underline{\vee} \sim q$ **c.** $\sim p \wedge q$
d. $\sim (p \wedge q)$

24. Let p and q be the statements:

p: Stevie Wonder is a famous singer.

q: Simon & Garfunkel is a famous law firm.

Determine the truth value of the following compound statements:

a. $p \wedge \sim q$ **b.** $p \underline{\vee} q$ **c.** $p \vee q$
d. $\sim (p \vee q)$

Note that p is true and q is false.

Solutions to Self-Help Exercises L.2

1.

p	q	r	$p \vee q$	$\sim q$	$r \wedge \sim q$	$(p \vee q) \vee (r \wedge \sim q)$
T	T	T	T	F	F	T
T	T	F	T	F	F	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	F	F	T
F	T	F	T	F	F	T
F	F	T	F	T	T	T
F	F	F	F	T	F	F

2. The statement p is true, while q is false. Thus

a. $\sim p$ is false. **b.** $p \vee q$ is true. **c.** $\sim p \wedge q$ is false.

d. $p \wedge \sim q$ is true is $\sim q$ is true.

e. $\sim p \vee \sim q$ is true.