Instructions:

- You must clear your calculator: MEM (2nd +), Reset (7), cursor right to ALL, All Memory (1), Reset (2).

- There are 25 questions and 14 pages to this exam including the cover sheet. True/False questions are worth 1 point each, the multiple choice questions are worth 4 points each, and the point values for the problems in the work out section are as indicated. There are 100 possible points.

- In order to receive full credit on the work out problems, you must show your work for each work out problem clearly and legibly. Include units on answers where appropriate.

- Box or circle your final answer in the work out problems.

- You may use a TI-83(+), TI-84(+), or TI-Nspire nonCAS version for all problems. Other calculators are not permitted.

- If you need extra scratch paper, ask me and I will provide you with some. Do NOT use your own papers.

- Your grade will be written on the second page of the exam.

Good Luck!
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Part 1: True or False. Clearly circle your answer. No work is necessary. No partial credit will be given. Each question is worth 1 point.

(1 pt.) 1. T  $\bigcirc$  If $x = a$ is a critical number of $f$, then $f(a)$ must be either a local maximum or a local minimum value of $f$.

(1 pt.) 2. T  $\bigcirc$  If $f$ is continuous near $a$ and $f'(a) = 0$ and $f''(a) > 0$, then $f$ has a local maximum at $a$.

(1 pt.) 3. (T)  \textbf{F}  If $f$ has an inflection point at $a$, then $f''(a) = 0$.

(1 pt.) 4. T  $\bigcirc$  A local maximum of $f$ only occurs at a point where $f'(x) = 0$.

(1 pt.) 5. (T)  \textbf{F}  The function $f(x) = x^2$ has a global minimum on any interval $[a, b]$.

(1 pt.) 6. T  $\bigcirc$  If $f$ is continuous on the interval $[a, b]$, then $\frac{d}{dx} \int_a^b f(x) \, dx = f(x)$.

(1 pt.) 7. T  $\bigcirc$  If $a < b$ and $\int_a^b f(x) \, dx = 0$, then $f(x) = 0$ for all $a < x < b$.

(1 pt.) 8. (T)  \textbf{F}  If $F(x)$ is an antiderivative of $f(x)$ and $G(x) = F(x) + 2$, then $G(x)$ is an antiderivative of $f(x)$.

(1 pt.) 9. T  $\bigcirc$  If $f$ is an even continuous function on $[-a, a]$, then $\int_{-a}^a f(x) \, dx = 0$.

(1 pt.) 10. (T)  \textbf{F}  Let $f$ be a continuous function on the interval $[a, b]$. There exist two constants $m$ and $M$ such that

$$m(b - a) \leq \int_a^b f(x) \, dx \leq M(b - a)$$
Part 2: Multiple Choice Questions: There are 10 multiple choice questions in this part and each question is worth 4 points. Indicate your answer clearly by marking the appropriate choice. If on a question more than one choice is marked, that question will be regarded as incorrect.

(4 pts.) 1. Which of the following is FALSE for a function $g$ if the second derivative of $g$ is the function whose graph is given below?

(a) $g$ has an inflection point at both $a = -1$ and $a = 2$.

b) $g$ is concave down on $(-1, 3)$.

c) $g$ is concave up on $(-2, -1)$.

d) $g'$ is decreasing on $(-1, 3)$.

e) $g'$ is increasing on $(-2, -1)$.
(4 pts.) 2. If \( f'(7) = 0 \) and \( f''(7) = -2 \) what can you say about \( f \)?

(a) \( f \) has a local maximum at \( x = 7 \).

b) \( f \) has a local minimum at \( x = 7 \).

c) \( f \) has an absolute maximum at \( x = 7 \).

d) \( f \) has an absolute minimum at \( x = 7 \).

e) \( f(7) = 0 \).

(4 pts.) 3. Which of the following is FALSE for the function \( h \) whose graph is given below.

\[ y = h(x) \]

\[ -2 -1 \quad 1 \quad 2 \]

\[ -2 \quad -1 \quad 1 \quad 2 \]

a) \( h \) attains its absolute minimum at an endpoint on \([-3, 3]\).

(b) At every point where \( h'(a) \) is zero or does not exist, \( h \) has a local maximum or minimum.

c) \( h \) has a local minimum at \( a = 0 \).

d) Every relative maximum and minimum of \( h \) occurs at a point where \( h'(a) \) is either zero or does not exist.

e) At \( a = -2.5 \) and \( a = 1 \), \( h \) has horizontal tangent line.
(4 pts.) 4. Which of the following integrals represents $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n} \cos \left( \frac{2i}{n} \right)$?

a) $\int_{2}^{4} \cos(x) \, dx$

b) $\int_{1}^{3} \cos(x) \, dx$

c) $\int_{0}^{2} \cos(x) \, dx$

d) $\int_{3}^{5} \cos(x) \, dx$

e) $\int_{4}^{6} \cos(x) \, dx$

(4 pts.) 5. Find $\int (\sec^2 t + \cos t) \, dt$

a) $\tan t - \sin t + C$

b) $\sec t + \sin t + C$

c) $\tan t + \sin t + C$

d) $\sec t - \sin t + C$

e) $-\tan t + \sin t + C$
(4 pts.) 6. For the function $f(t)$ whose graph is given in the figure below, what is the value of $\int_1^5 f(t) \, dt$?

![Graph of a function $f(t)$]

a) 7  
b) 5  
c) 0  
(d) 3  
e) 2

(4 pts.) 7. The velocity of an object traveling on a straight line is $v(t) = \frac{3}{2}t^{1/2} - t^{-1}$. If $s(t)$ denotes the position at time $t$ and $s(1) = e^{3/2}$, find $s(1)$.

a) $s(1) = 2 + e^{-2}$  
(b) $s(1) = 2$  
c) $s(1) = 2 + 2e^{3/2}$  
d) $s(1) = 2e^{3/2}$  
e) $s(1) = e^2$
8. Evaluate $\int_1^2 (e^x + \frac{1}{x}) \, dx$ to four decimal places.

a) $2.8756$

b) $3.7654$

c) $-2.2156$

d) $5.3639$

e) $2.2156$

9. Water is pouring out of a pipe at the rate of $f(t)$ gallons/minute. You collect the water that flows from the pipe between $t = 2$ minutes and $t = 4$ minutes. The amount of water you collect can be represented by:

a) $f'(4) - f'(2)$

b) $\int_2^4 f'(t) \, dt$

c) $\int_2^4 f(t) \, dt$

d) $f(4) - f(2)$

e) $f(4)$

10. If $f(x) = \int_x^2 (t^3 + 2t^2) \, dt$ find $f''(x)$.

a) $3x^2 + 4x$

b) $-3x^2 - 4x$

c) $x^3 + 2x^2$

d) $-x^3 + 2x^2$

e) $-x^3 - 2x^2$
Part 3: Work out Problems. There are 5 problems in this part. For each of them write a complete solution, and show your work clearly and legibly. Include units on answers where appropriate.

(8 pts.) 1. Use calculus to find the absolute maximum and absolute minimum of the function \( f(x) = x^3 - 6x^2 + 9x + 2 \) on the interval \([-1, 4]\).

\[
\begin{align*}
\frac{df}{dx} &= 3x^2 - 12x + 9 = 3(x - 1)(x - 3) \\
\frac{df}{dx} = 0 &\iff x = 1 \text{ or } x = 3 \quad \text{critical numbers.} \\
f(1) &= 1 - 6 + 9 + 2 = 6 \quad \text{Absolute maximum.} \\
f(3) &= 3^3 - 6 \cdot 3^2 + 9 \cdot 3 + 2 = 2 \\
f(-1) &= (-1)^3 - 6 \cdot (-1) + 9 \cdot (-1) + 2 = -14 \quad \text{Absolute minimum} \\
f(4) &= 4^3 - 6 \cdot 4^2 + 9 \cdot 4 + 2 = 6 \quad \text{Absolute maximum}
\end{align*}
\]
(7 pts.) 2. The acceleration of an object moving on a straight line is given by 
\( a(t) = 6t - 4t^2 \). Let \( s(t) \) be its position at time \( t \) and \( v(t) \) be its velocity at time \( t \). If we know that \( s(0) = 4 \) and the object is standing still at \( t = 1 \), find \( s(t) \).

\[
v(t) = \int a(t) \, dt = \int (6t - 4t^2) \, dt = 3t^2 - \frac{4}{3}t^3 + C
\]

\( 0 = v(1) = 3 - \frac{4}{3} + C \quad \Rightarrow \quad C = -\frac{5}{3} \)

\[
s(t) = \int v(t) \, dt = \int (3t^2 - \frac{4}{3}t^3 - \frac{5}{3}) \, dt = t^3 - \frac{1}{3}t^4 - \frac{5}{3}t + C
\]

\( 4 = s(0) = C \)

\[
s(t) = t^3 - \frac{t^4}{3} - \frac{5}{3}t + 4
\]
4. A piece of wire 10 meters long is cut into at most two pieces. One piece is bent into a square and the other is bent into a circle. How should the wire be cut so that the total area enclosed is a maximum?

\[ 4x + 2\pi r = 10 \Rightarrow r = \frac{10-4x}{2\pi} = \frac{5-2x}{\pi} \]

\[ A = x^2 + \pi r^2 = x^2 + \pi \left( \frac{5-2x}{\pi} \right)^2 = x^2 + \frac{1}{\pi} (5-2x)^2 \]

Maximize \[ A(x) = x^2 + \frac{1}{\pi} (5-2x)^2 \] on \[ [0, \frac{5}{2}] \].

\[ A'(x) = 2x + \frac{2}{\pi} (5-2x) (-2) = 2x + \frac{8x}{\pi} - \frac{20}{\pi} = \frac{(2\pi+8)x - 20}{\pi} \]

\[ A'(x) = 0 \iff x = \frac{10}{\pi+8} \approx 1.4 \]

\[ A(1.4) = (1.4)^2 + \frac{1}{\pi} (5-2(1.4))^2 \approx 3.5 \]

\[ A(0) = \frac{25}{\pi} \approx 7.95 \leftarrow \text{maximum} \]

\[ A\left(\frac{5}{2}\right) = 6.25 \]

The wire should be used to make only a circle without a cut.
(10 pts.) 3. Evaluate the following integrals.

a) \[ \int xe^{x^2} \, dx = \frac{1}{2} \int e^{u} \, du = \frac{1}{2} e^{u} + C = \frac{1}{2} e^{x^2} + C \]
\[ u = x^2 \]
\[ du = 2x \, dx \]

b) \[ \int_{-\pi/4}^{\pi/4} \frac{\sin x}{\cos^2 x} \, dx = 0 \quad \text{since the integrand is an odd function.} \]

Alternatively, by the substitution method,
\[ u = \cos x \quad \Rightarrow \quad du = -\sin x \, dx \]
\[ \int_{\pi/4}^{\pi/4} \frac{\sin x}{\cos^2 x} \, dx = \int_{1/\sqrt{2}}^{1/\sqrt{2}} u^{-2} \, du = 0 \]
(15 pts) 5. Let \( f(x) = (x - 2)(x - 1)^2 = x^3 - 4x^2 + 5x - 2 \).

(a) What is the domain of \( f \)?
\[
\text{Dom}(f) = (-\infty, \infty)
\]

(b) What are the zeros of \( f \)?
\[
f(x) = (x - 2)(x - 1)^2 = 0 \iff x = 2 \quad \text{or} \quad x = 1 \quad \text{(double)}
\]

(c) What are the critical numbers of \( f \)?
\[
f'(x) = 3x^2 - 8x + 5 = (3x - 5)(x - 1) = 0 \iff x = \frac{5}{3} \quad \text{or} \quad x = 1
\]

(d) On which intervals \( f \) is increasing? On which intervals \( f \) is decreasing?
\[
\begin{array}{cccc}
& 1 & & \frac{5}{3} \\
\hline
f' & + & 0 & - & 0 & + \\
f \quad \text{increasing on} & (-\infty, 1) \cup \left(\frac{5}{3}, \infty\right)
\end{array}
\]
\[
f \quad \text{decreasing on} \quad \left(1, \frac{5}{3}\right)
\]
(e) Find the local maximum and minimum values of $f$.

$$f''(x) = 6x - 8$$

$$f'(1) = -2 < 0 \implies x = 1 \text{ is a local max, } f(1) = 0$$

$$f''\left(\frac{5}{3}\right) = 2 > 0 \implies x = \frac{5}{3} \text{ is a local min, } f\left(\frac{5}{3}\right) = -\frac{4}{2.7} \approx -0.15$$

(f) Find the intervals of concavity and the inflection points.

$$f''(x) = 6x - 8 = 0 \iff x = \frac{4}{3} \text{ is an inflection point. } f\left(\frac{4}{3}\right) \approx -0.07$$

$$f''$$

$$\begin{array}{c|c|c}
\text{Interval} & f'' & \text{Concavity} \\
\hline
\left(-\infty, \frac{4}{3}\right) & - & \text{down} \\
\left(\frac{4}{3}, \infty\right) & + & \text{up}
\end{array}$$

(g) Use the information from parts (a)-(f) to sketch the graph of $f$ on the grid below. (Do NOT use a graphing calculator.)