3.3 Derivatives of Trigonometric Functions

**Preview Activity 1:** Consider the function \( f(x) = \sin(x) \), which is graphed below. Note carefully that the grid in the diagram does not have boxes that are 1 \( \times \) 1, but rather approximately 1.57 \( \times \) 1, as the horizontal scale of the grid is \( \pi/2 \) units per box.

(a) At each of \( x = -2\pi, -\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, 2\pi \), use a straightedge to sketch an accurate tangent line to \( y = f(x) \).

(b) Use the provided grid to estimate the slope of the tangent line you drew at each point. Pay careful attention to the scale of the grid.

(c) Estimate \( f'(0) \) by using small values of \( h \), and compare the result to your visual estimate for the slope of the tangent line to \( y = f(x) \) at \( x = 0 \) in (b). Using periodicity, what does this result suggest about \( f'(2\pi) \)? about \( f'(-2\pi) \)?

(d) Based on your work in (a), (b), and (c), sketch an accurate graph of \( y = f'(x) \) on the grid given at the bottom.

(e) What familiar function do you think is the derivative of \( f(x) = \sin(x) \)?
**Preview Activity 2:** Consider the function \( g(x) = \cos(x) \), which is graphed below. Note carefully that the grid in the diagram does not have boxes that are 1 × 1, but rather approximately 1.57 × 1, as the horizontal scale of the grid is \( \pi/2 \) units per box.

(a) At each of \( x = -2\pi, -\frac{3\pi}{2}, -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, 2\pi \), use a straightedge to sketch an accurate tangent line to \( y = g(x) \).

(b) Use the provided grid to estimate the slope of the tangent line you drew at each point. Pay careful attention to the scale of the grid.

(c) Estimate \( g'(\frac{\pi}{2}) \) by using small values of \( h \), and compare the result to your visual estimate for the slope of the tangent line to \( y = g(x) \) at \( x = \frac{\pi}{2} \) in (b). Using periodicity, what does this result suggest about \( g'(\frac{3\pi}{2}) \)? Can symmetry on the graph help you estimate other slopes easily?

(d) Based on your work in (a), (b), and (c), sketch an accurate graph of \( y = g'(x) \) on the grid given at the bottom.

(e) What familiar function do you think is the derivative of \( g(x) = \cos(x) \)?
Activity 1: Use the quotient rule to derive formulas for the trigonometric functions:

(a) \( \frac{d}{dx}(\tan x) = \)

(b) \( \frac{d}{dx}(\cot x) = \)

(c) \( \frac{d}{dx}(\sec x) = \)

(d) \( \frac{d}{dx}(\csc x) = \)
**Activity 2:** Find an equation of the tangent line to the curve $y = x \sin x$ at the point $(\pi/2, \pi/2)$

**Activity 3:** Differentiate

(a) $y = \frac{1 + \sin x}{x + \cos x}$ at the point $(\pi/2, \pi/2)$

(b) $y = e^x \csc x$