5.3 Evaluating Definite Integral

In the previous section we saw how to compute definite integrals using limits and Riemann sums. In this section we will see how one can compute definite integrals in much easier way using the following theorem:

**Evaluation Theorem:** If \( f \) is continuous on the interval \( [a, b] \), then

\[
\int_a^b f(x)dx = F(b) - F(a)
\]

where \( F \) is any antiderivative of \( f \), that is \( F' = f \).

**Note:** We will sometimes use the notation \( F(x)\bigg|_a^b = F(b) - F(a) \)

**Activity 1:** Evaluate the following integrals.

(a) \( \int_0^1 x^2 dx \)

(b) \( \int_1^2 e^x dx \)
(c) \[ \int_0^3 (2e^{-y} + 4 \sin y)\,dy \]

(d) \[ \int_0^{\pi/4} \sec^2(\theta)\,d\theta \]

(e) \[ \int_0^2 |2x - 1|\,dx \]
Indefinite Integrals

Recall that $\int f(x)dx$ denotes the general antiderivative of $f$, that is $\int f(x)dx = F(x) + C$ where $F'(x) = f(x)$, and we call the symbol $\int f(x)dx$ the indefinite integral of $f$.

Important Remark: You should distinguish carefully between definite and indefinite integrals. A definite integral $\int_a^b f(x)dx$ is a number, whereas the indefinite integral $\int f(x)dx$ is a function.

Activity 2: Find

(a) $\int (6 - 4x^2)dx$

(b) $\int (\cos x + \frac{1}{x+1})dx$
**The Net Change Theorem:** The integral of a rate of change is the net change

\[ \int_{a}^{b} F'(x) \, dx = F(b) - F(a) \]

**Activity 3:** A honeybee population starts with 100 bees and increases at a rate of \( n'(t) \) bees per week. What does \( 100 + \int_{0}^{15} n'(t) \, dt \) represent?

**Activity 4:** Water flows from the bottom of a storage tank at a rate of \( r(t) = 200 - 4t \) liter per minute, where \( 0 \leq t \leq 50 \). Find the amount of water that flows from the tank during the first 10 minutes.

**Activity 5:** The velocity function (in meters per second) \( v(t) = t^2 - 2t - 8 \), \( 1 \leq t \leq 6 \), is given for a particle moving along a line. Find

(a) the displacement.

(b) the distance traveled by the particle during the given time interval.
5.4 The Fundamental Theorem of Calculus

Preview Activity 1: Let \( g(x) = \int_{1}^{x} f(t) \, dt \) where \( f \) is the function whose graph is given below.

(a) Find \( g(0), g(1), g(2), g(3) \) and \( g(6) \).

(b) On what intervals is \( g \) increasing? Where does it have a maximum?

(c) Sketch a rough graph of \( g \).
Preview Activity 2: Let \( g(x) = \int_1^x t^3 \, dt \).

(a) Find a formula for \( g(x) \).

(b) What does your answer represent?

(c) Find \( g'(x) \).

Fundamental Theorem of Calculus: Suppose \( f \) is continuous on \([a, b]\).

1. If \( g(x) = \int_a^x f(t) \, dt \) then \( g'(x) = f(x) \) for \( a < x < b \). That is \( g(x) \) is an antiderivative of \( f \).

2. \( \int_a^b f(x) \, dx = F(b) - F(a) \), where \( F \) is any antiderivative of \( f \), that is, \( F' = f \). (Observe that this is just the evaluation theorem from the previous section).
Activity 1: Use Part I of the Fundamental Theorem of Calculus to find the derivative of the following functions.

(a) $g(x) = \int_{0}^{x} \sqrt{1 + t^2} dt$

(b) $g(x) = \int_{1}^{x} \frac{1}{t^5} dt$

(c) $g(x) = \int_{1}^{x^4} \sec t \, dt$

(d) $g(x) = \int_{x}^{5} \sqrt{1 + t} \, dt$

(e) $g(x) = \int_{e^x}^{0} \sin^3 u \, du$

(f) $g(x) = \int_{-x}^{x^2} (1 + t^3) \, dt$