2.6 Derivatives and Rates of Change

**Preview Activity:** Suppose that $f$ is the function given by the graph below and that $a$ and $a + h$ are the input values as labeled on the $x$-axis. Use the below graph to answer the following questions.

(a) Locate and label the points $(a, f(a))$ and $(a + h, f(a + h))$ on the graph.

(b) Construct a right triangle whose hypotenuse is the line segment from $(a, f(a))$ to $(a + h, f(a + h))$. What are the lengths of the respective legs of this triangle?

(c) What is the slope of the line that connects the points $(a, f(a))$ and $(a + h, f(a + h))$?

(d) Write a meaningful sentence that explains how the average rate of change of the function on a given interval and the slope of a related line are connected.

(e) What happens to the line that connects the points $(a, f(a))$ and $(a + h, f(a + h))$ as $h$ gets smaller and smaller?

(f) Conjecture a formula for the slope of the line that touches the graph of $f$ at the point $(a, f(a))$?
**Definition:** The tangent line to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through $P$ with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided that this limit exists.

**Activity 1:** Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

**Activity 2:** Recall that in Section 2.1, we tried to compute the instantaneous velocity of a falling ball whose position function given as $f(t) = 64 - 16(t - 1)^2$.

(a) Write a formula that gives the instantaneous velocity of the ball at time $t = a$ in terms of the tangent lines.

(b) Use your formula to compute the instantaneous velocity at time $t = 2$. Compare your results with the previous ones.

(c) How fast is the ball traveling when it hits the ground?