There will be three parts in Exam II: one of True/False questions, one for multiple choice problems, and a section of work out problems. It will cover Sections 2.6-2.8, 3.1-3.4, 3.7-3.9. Here are the concepts you are expected to know:

Definitions and Theorems You Should Know:

1. The limit definition of the derivative of $f$ at the point $a$

$$f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

2. An alternative limit definition of the derivative of $f$ at the point $a$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

3. For any $n \in \mathbb{R}$, the Power Rule states that

$$\frac{d}{dx} x^n = nx^{n-1}$$

4. If $f$ and $g$ are both differentiable, then the Product Rule states that

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx}g(x) + g(x) \frac{d}{dx}f(x)$$

5. If $f$ and $g$ are both differentiable, then the Quotient Rule states that

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx}f(x) - f(x) \frac{d}{dx}g(x)}{g(x)^2}$$

6. If $g$ is differentiable at $x$ and $f$ is differentiable at $g(x)$, then the Chain Rule states that the composite function $f \circ g$ is differentiable at $x$ and

$$(f \circ g)'(x) = f'(g(x))g'(x)$$

7. A function $f$ is differentiable at $a$ if $f'(a)$ exists (that is, the limit defined in 1 or 2 exists).

8. A function $f$ is differentiable on an interval $(a, b)$ if $f$ is differentiable at every point in that interval.

9. The Linearization of a function $f$ at a point $a$ is the function

$$L(x) = f(a) + f'(a)(x - a)$$
Other Topics:

- Know how to compute derivatives from the limit definitions.
- Know how to find an equation of the tangent line to a curve at a given point.
- Know Point-Slope Form equation for finding the equation of a line between two given points.
- Know how differentiability and continuity are related (see Section 2.7).
- Know how to determine properties of $f$ from the graphs of $f'$ and $f''$.
- Know how to determine properties of $f$ from its graph (i.e. increasing, decreasing, concavity, etc.)
- Know how to draw graphs of functions with certain derivative and second derivative properties (as in Section 2.8).
- Know the definitions of position, velocity and acceleration and how they relate to one another.
- Know all differentiation rules in Section 3.1.
- Know properties of logarithms and exponential functions, and how they relate to one another (including cancellation equations)
- Know Product, Quotient, and Chain Rules and how to compute derivatives using them.
- Know derivatives of all basic functions.
- Know derivatives of Trigonometric functions (see Section 3.3)
- Know how to apply differentiation techniques to solve real world type problems (see Section 3.8)
- Know how to form the Linearization of a function $f$ at a point $a$, and how to estimate a function using this.
- Know how to use Differentials to approximate error.
Example Problems:

1. Find the derivative of $f(x) = x^2 + \sqrt{x}$ using the limit definition of derivative.

2. Differentiate $y = \frac{2x^2 + 6x + 2}{\sqrt{x}}$. 

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3. Find the equation of the tangent line to the curve \( f(x) = \ln \left( \frac{x^2 + 1}{\tan(\pi x)} \right) \) at the point \( x = 1/4 \).

4. Find the points on the curve \( y = 2x^3 + 5x^2 - 4x + 8 \) where the tangent is horizontal.
5. Differentiate

(a) $f(x) = \frac{3 - xe^x}{x + e^x}$.

(b) $y = \cos(a^4 + x^4)$. 
6. The graph of the derivative of a continuous function $f$ is shown.

(a) On what intervals $f$ is increasing? Decreasing?

(b) At what values of $x$ does $f$ have a local maximum? Local minimum?

(c) On what intervals $f$ is concave upward? Concave downward?

(d) State the $x$-coordinate(s) of the point(s) of inflection.

(e) Assuming that $f(0) = 0$, sketch a graph of $f$. 
7. If \( y = 3x^2 \cos x \cot x \), find \( \frac{dy}{dx} \).
8. Consider $f(x) = 5 + 12x - x^3$. Find

(a) the intervals on which $f$ is increasing

(b) the intervals on which $f$ is decreasing

(c) the open intervals on which $f$ is concave up

(d) the open intervals on which $f$ is concave down

(e) the x-coordinates of all inflection points.
9. The equation of motion of a ball is \( s = 27t - t^3 \), where \( s \) is in meters and \( t \) is in seconds.

(a) Find the maximum height of the ball.

(b) When does the ball hit the ground?

(c) What is the velocity of the ball when it hits the ground?

(d) Find the acceleration when the velocity is 0.
10. (a) Find the linearization of the function $f(x) = x^{3/4}$ at the point $a = 16$.

(b) Use your answer from part (a) to estimate the value of $(16.1)^{3/4}$. 