Fanny Dos Reis

Personal

Office Address:	Milner 010 Department of Mathematics Texas A&M University 3368 TAMU College Station, Texas 77843-3368
Work phone:	845-3629
e-mail:	`fdr@math.tamu.edu`
Web page:	`http://www.math.tamu.edu/~fdr/`

Current position

2006-2007 Visiting Assistant Professor

Education, diplomas

12/18/2003 Ph.D. in Mathematics (Symplectic Geometry) from École Normale Supérieure de Lyon, France.
Advisor: Prof. J.-C. Sikorav
(`jean-claude.sikorav@umpa.ens-lyon.fr`).
Title: *Pseudoholomorphic curves and currents in an almost complex manifold* ("Courbes pseudoholomorphes et courants dans une variété presque complexe").

1998-1999

- M.Sc. in Mathematics and Computer Science (Magistère de Mathématiques et d'Informatique), École Normale Supérieure de Cachan–Université Paris VII, France.
- Diplôme d'Études Approfondies de Mathématiques, Université Paris VII, France.
  Thesis: *Weierstrass representation and spinorial technics* ("Représentation de Weierstrass et techniques spinorielles").
  Supervisor: Prof. F. Helein (`helein@math.jussieu.fr`)
- French Certificate of University Teaching in Mathematics (*Agrégation de Mathématiques*).

1996-1998 Licence et Maîtrise de Mathématiques, Université Paris VII–ENS de Cachan, France.
Degree thesis: *Morse Theory* ("Théorie de Morse").
Supervisor: Prof. H. Rosenberg (`rosen@math.jussieu.fr`).

1996-1999 Student at École Normale Supérieure de Cachan, France.

1993-1996 Mathématiques Supérieures and Mathématiques Spéciales at Lycée H. Wallon, Valenciennes, France.

Teaching experience

2006-2007 Lectures in Mathematics, at undergraduate level, Texas A&M University.

2004-2006 Lectures in Pure Mathematics, at undergraduate level, Université de Lille 1, France.

2003-2004 Practical classes in Pure Mathematics, at undergraduate level, Université Paris XI, France.

2000-2003 Practical classes in Applied Mathematics, at undergraduate level, at INSA de Lyon, France (Engineering University); EURINSA branch (For French and European students).

Invited lectures

01/2007 *Results on the regularity of E-cycles* ("Resultats sur la regularité des E-cycles"), Université de Lille 1, France.

10/2006 *On the regularity of elliptic cycles*, Texas A&M University, College Station, Texas.

06/2005 *Pseudoholomorphic curves and currents* ("Courbes pseudoholomorphes et courants"), Université de Lyon 1, France.

04/2005 *Characterization of J-holomorphic curves, using rectifiable currents* ("Une caractérisation des courbes J-holomorphes par des courants rectifiables"), École Polytechnique, France.

02/2005 *Pseudoholomorphic curves and currents* ("Courbes pseudoholomorphes et courants"), Université de Marseille 1, France.

02/2005 *Pseudoholomorphic curves and currents*, Texas A&M University, College Station, Texas.

02/2004 *J-holomorphic curves and currents* ("Courbes J-holomorphes et courants"), Université de Lille 1, France.

02/2002 *King's theorem in four dimensional almost complex manifolds* ("Théorème de King pour les variétés presque-complexes de dimension 4"), Université de Lille 1, France.

School and congress attended

09/2003 Summer school: Symplectic 4-manifolds and algebraic surfaces. C.I.M.E. ("*International Mathematical Summer Center*") course, Cetraro, Italy.

06/2002 Summer school: Topological methods in complex analysis and geometry ("*Méthodes topologiques en analyse complexe et géométrie*"), Université de Lille 1, France.

06/2002 Congress: Almost complex geometry and symplectic topology ("*Géométrie presque complexe et topologie symplectique*"), Université de Lille 1, France.

07/2001 Summer school: Symplectic geometry ("*Géométrie symplectique*"), Université de Paris VII, France.

References

J.-C. Sikorav	:	`jean-claude.sikorav@umpa.ens-lyon.fr`
F. Hélein	:	`helein@math.jussieu.fr`
H. Rosenberg	:	`rosen@math.jussieu.fr`

Teaching Statement

I consider that my teaching mission is to guide students during their education, to enable them to carry out their own educational project. I have been teaching Mathematics in several universities for 7 years. In France, I had been in charge of lecture of Algebra (complex numbers, polynomials, vector spaces, matrices, some Logic notions, Arithmetic, Affine Geometry) and Analysis (continuity, inverse functions, Taylor formula, sequences, integration, fonctions of two variables) at an engineering university (INSA) in Lyon, in Paris XI University (Paris Sud, Orsay), and in Lille 1 university. This academic year, I taught Calculus for business and for Engineers at Texas A&M University.

I have always followed my principle:

I fit my teaching to the students' expectations and levels.

While I was working in Lyon, most of my students wanted to become engineers. None of them wanted to be a mathematician. So, I emphasized Applied Mathematics rather than Pure Mathematics. I usually encouraged them to use their calculators to get intuitions of the results or to check they were right. I managed several Computer Algebra System (Maple) courses. For example, they draw the graph of Taylor approximation polynomials of cosinus.

In Lille, most of my students want to become teachers in mathematics. So I stress exercices that could help them to pass the competition for High School Teaching.

In Lyon, the students came from several European countries. They did not have the same background. In addition of the practice classes, I was in charge of support course: I answered all the questions about what they had not learnt in their countries, what they did not understand in the lectures or in my classes. I gave them basic exercices to help them solve their problems. It requires all my attention to give them an efficient help: see where the problem lies, find simple examples or simple exercices to make them understand. I think that I succeeded because they kept asking questions during the semester.

I try to be as available as possible:

- I do grade any works they have done by themselves and they want me to check.
- I meet students during the office hours and appointment.
- Most of the time, I answered their e-mails within at most a day. I do not mind spending time answering their questions.

Finally, I inform them about scientific events, such as public conferences, scientific films, open days in universities,... I participated in the open days of the École Normale Supérieure de Lyon. Once, I gave a talk about Pythagoras' and Bolyai's theorems. Another year, I brought material (regular polygons and glue) to construct with the visitors all the Plato's polyhedrons.

Reseach Statement

During my Ph.D. [DRL], I had investigated the relationship between J-holomorphic curves on one hand, and closed, positive, locally rectifiable currents of type $(1,1)$ on the other hand. This problem deals with the generalization of Plateau's problem in dimension greater than 3. A way to attack this problem is to introduce a complex structure and notice that complex submanifolds are minimizing. In this case, J. R. King [K] and R. Harvey [H] showed that a closed, positive, locally rectifiable current of type $(1,1)$ is a positive holomorphic chain. One of the goals of my Ph.D. was to generalize that theorem to an almost complex manifold. The proof of J. K. King's theorem depends on Lelong Formula. It cannot be easily adapted to the almost complex case because the ∂^2 operator is no more null and the analysis of the problem becomes much more complicated. But, J-holomorphic curves satisfy elliptic equations. It implies that the regularity results are the same as in the complex case. The currents I considered satisfy a quasiminimising property which is not far from the complex case.

In dimension 4, Taubes [T] showed that a sequence of currents coming from the Seiberg–Witten equations converges to a closed, positive, locally rectifiable current of type $(1,1)$. Then he tried to show that the limit is a J-holomorphic curve. But his proof contained a serious gap in the study of the singular points. My work partially filled the gap in Taubes's results.

I showed that in an almost complex manifold, a closed, positive locally rectifiable current of type $(1,1)$ converges in flat norm to his tangent cone. That generalizes B. White's result [W] on tangent cone of minimizing current. I showed that in a 4-dimensional manifold, such a current is locally a J-holomorphic curve in the neighborhood of most of the points of the support.

At the same time, independantly of my work, in October 2003, T. Rivière and G. Tian [RT] published a result to the effect that a closed locally rectifiable current of type $(1,1)$, compatible with a symplectic form is a J-holomorphic curve. Their result then appear more general than mine: in dimension 4, all the almost complex structures are locally compatible. Since the methods they used in their paper are essentially the same as mine, I could not reasonably publish a paper about that result I obtained, sadly.

I have several directions for my future research.

On one hand, I would like to generalize King's theorem in the following three directions:

- I would like to investigate closed positive locally rectifiable currents of type (p,p) in particular, their singular sets. I would like to show that their singular sets are composed by almost complex submanifolds of positive codimensions. That would be a nice result, even if rectifiable currents of type (p,p) $(p>1)$ and almost complex submanifolds of real dimension greater than 2 only exist for *non generic* almost complex structures. The problem indeed is overdetermined.

- Another question is: can we generalize Siu's theorem? What about closed, positive currents of type $(1,1)$ (without the rectifiability condition)? Taubes method cannot be adapted because it is based on integer

Lelong number.

- Elliptic structures were introduced by M. Gromov [G]. They generalise almost complex structures in dimension 4. They are “non linear” complex structures. I am currently working on the proof of the following result: in a 4-dimensional manifold with an elliptic structure E, locally rectifiable elliptic currents are E-curves. For the moment, I have shown that such a current is locally the graph of a Hölder multivalued function. In a open dense set (the regular points), it is locally an embedded E-disk. If the singular set is isolated points or limit of isolated points, the current is locally an E-disk. I still have to find a reason why the singular set is only composed of isolated points. In the almost complex case, the reason is the consequence of a quasiminimizing property. Elliptic currents do not satisfy this condition.

On the other hand, the results contained in my Ph.D thesis give a new tool to study the modular space of J-holomorphic curves. The classical approach to study J-holomophic curves is to consider a Riemann surface and a J-holomorphic map. Using currents enables us to make the genus of the Riemann surfaces irrelevant. I may study the whole modular space instead of studying its restrictions to riemann surfaces with a fixed genus. A first step would be to show a sort of upper-semi-continuity of the genus.

Article in preparation

- Fanny Dos Reis. *On the regularity of E-curves.* In preparation.

References

[DRL] Fanny Dos Reis-Lévecque. *Courbes pseudoholomorphes et courants.* PhD thesis, École Nomale Supérieure de Lyon, http://math.univ-lille1.fr/~levecque/, Décembre 2003.

[G] Michael Gromov. Pseudoholomorphic curves in symplectic manifolds. *Invent. Math.*, 82(2):307–347, 1985.

[H] Reese Harvey. Holomorphic chains and their boundaries. In *Several complex variables (Proc. Sympos. Pure Math., Vol. XXX, Part 1, Williams Coll., Williamstown, Mass., 1975)*, pages 309–382. Amer. Math. Soc., Providence, R. I., 1977.

[K] James R. King. The currents defined by analytic varieties. *Acta Math.*, 127(3-4):185–220, 1971.

[RT] Tristan Rivière and Gang Tian. The singular set of 1-1 integral currents. `http://arxiv.org/pdf/math.AP/0310052`.

[T] Clifford H. Taubes. SW⇒Gr: from the Seiberg-Witten equations to pseudo-holomorphic curves. *J. Amer. Math. Soc.*, 9(3):845–918, 1996.

[W] Brian White. Tangent cones to two-dimensional area-minimizing integral currents are unique. *Duke Math. J.*, 50(1):143–160, 1983.